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ON A BIVARIATE PROBABILITY MODEL FOR NUMBER OF CONCEPTIONS CONSIDERING SECONDARY STERILITY

U. Srivastava* & B.N. Bhattacharya**

**Mahila Maha vidyalaya, Banaras Hindu University, Varanasi, India.*

***Ex- Professor, Indian Statistical Institute, Kolkata, India.*

Abstract

In this paper a bivariate probability model for two types of conceptions occurring during the latter phase of the reproductive span of females has been proposed when the start of the observational period is a distant point since marriage. The model has been formulated by considering the incidence of secondary sterility during the observational period. The model is applied on a real data for illustration.

Introduction

A univariate probability distribution for number of conceptions during a given period of time is derived by Srivastava and Singh (1989) in which variation in sterility parameter with respect to age is considered. Singh (1978) proposed and applied a bivariate probability model during any segment of the reproductive span, the starting point being a distant one since marriage, for two types of conception, viz., those resulting in live birth and the offspring continuing to live beyond the period of one year, and those in which the offspring dies within the period of one year.

Implicit, however, in Singh's study. It is considered that the female, who is fecund at the start of the observational period, remains so throughout the period. But Srivastava and Singh (1989) have considered that transition from fecundity to sterility is age-dependent and the rate of transition increases, especially in the latter segment of marital life. Hence Singh's model is not suitable when applied for the reproductive performance of females for the later period of reproductive span.

Keeping the above in view, we have modified Singh's model by considering the incidence of secondary sterility during the observational period. The present paper deals with the formulation of a probability distribution employing observation on the number of live birth conceptions of two types, viz., child dying within a year and child surviving for more than a year. The model is elaborated along with the assumptions in Section 2. Additionally, the model has been applied to an observed distribution, for illustration, taken from the survey. "Rural Development and Population Growth (1978)". Section 3 deals with the application. The conclusion follow in Section 4.

Model

A female is observed between time points t_1 and t_2 ($t_1 < t_2$ and $t_2 - t_1 = T$), where t_1 is a distant point since marriage. Let us assume the following :

1. The female has led a married life throughout the observational period (t_1, t_2).
2. The probability that a female fecund at t and conceiving during the interval $(t, t + \Delta t)$ is $m \Delta t + O(\Delta t)$; $t_1 < t < t_2$ and $m > 0$.
3. Each conception results in a live birth. Further let $(1-\theta)$ be the probability that a child born alive, dies before completing the first year of life (taken here as type I conception) and θ be the probability that the child survives for more than a year (taken here as type II conception).
4. Let h_1 and h_2, ($h_1 < h_2$) be the rest periods associated with the conceptions of types I and II respectively.
5. Further, the above-mentioned assumptions are also true for a considerable period of time prior to t_1 so that the reproduction process is in equilibrium at t_1.
6. Let a_1 be the probability that the female is fecund at the start of the observational period. Then it follows that $(1 - a_1)$ is the probability that she is incapable of producing children during the observational period.
7. Let $(1 - a_2)$ be the probability that the female becomes sterile during the interval (t_1, t_2), given that she was fecund at the start of the observation period. Further, the probability that such a female will become sterile during $(t, t + dt)$ is $g(t)\,dt$ where $g(t) = \frac{1}{T}$, $t_1 < t < t_2$

Let $X_1(\tau)$ and $X_2(\tau)$ denote the number of conceptions of types I and II respectively to a female during a time interval $(t_1, t_1 + \tau)$ of length τ $(0 < \tau \leq t_2 - t_1)$.

Let $P[X_1(\tau) = i, X_2(\tau) = j] = P_{i,j}(\tau)$ denotes the joint probability function of $X_1(\tau)$ and

$X_2(\tau)$.

Under the assumption 4, the maximum number of conceptions of type I to a female during the period of observation of length T cannot exceed n_1, while in case of conceptions of type II, this number is n_2 where $n_i \le [T/h_i] + 1$, $[T/h_i]$ stands for the greatest integer not exceeding T/h_i for i= 1, 2.

Let the successive conceptions occur at times T_1, $T_1 + T_2$, $T_1 + T_2 + T_3$, …, measured from the point t_1 . T_1 is the time of first recording of conception and T_r $(r > 1)$ is the time between (r-1)th and r^{th} recordings. Under the assumptions 1 to 5, the probability density function (p.d.f.) of T_1 is given by

$$\begin{aligned} f_1(t) &= \frac{1}{\mu}, && 0 < t \le h_1 \\ &= \frac{1}{\mu} - \frac{(1-\theta)}{\mu}\left(1 - e^{-m(t-h_1)}\right), && h_1 < t \le h_2 \\ &= \frac{1}{\mu} - \frac{(1-\theta)}{\mu}\left(1 - e^{-m(t-h_1)}\right) - \frac{\theta}{\mu}\left(1 - e^{-m(t-h_1)}\right), && t > h_2 \end{aligned} \quad (1)$$

Further, the p.d.f. of the waiting time of the (i+j)th conception since t_1, given that, among the preceding (i+j-1) conceptions, (i-1) are of type I and j are of type II, is given by (Singh, 1978)

$$\begin{aligned} f_{i+j}(t/\, i-1, j) &= 0; && 0 < t \le (i-1)h_1 + jh_2 \\ &= \pi_{1,1}(t, i, j); && (i-1)h_1 + jh_2 < t \le ih_1 + jh_2 \\ &= \pi_{1,1}(t, i, j) - \pi_{1,2}(t, i, j); && ih_1 + jh_2 < t \le (i-1)h_1 + (j+1)h_2 \\ &= \pi_{1,1}(t, i, j) - \pi_{1,2}(t, i, j) - \pi_{1,3}(t, i, j); && t > (i-1)h_1 + (j+1)h_2 \end{aligned} \quad (2)$$

Where,

$$\pi_{1,1}(t, i, j) = \frac{1}{\mu}\left[1 - e^{-m\left(t-\overline{i-1}h_1-jh_2\right)} \sum_{s=0}^{i+j-2} \frac{m^s\left(t-\overline{i-1}h_1-jh_2\right)^s}{s\,!}\right]$$

$$\pi_{1,2}(t, i, j) = \frac{(1-\theta)}{\mu}\left[1 - e^{-m(t-ih_1-jh_2)} \sum_{s=0}^{i+j-1} \frac{m^s(t-ih_1-jh_2)^s}{s\,!}\right]$$

and

$$\pi_{1,3}(t, i, j) = \frac{\theta}{\mu}\left[1 - e^{-m\left(t-\overline{i-1}h_1-\overline{j+1}h_2\right)} \sum_{s=0}^{i+j-1} \frac{m^s\left(t-\overline{i-1}h_1-\overline{j+1}h_2\right)^s}{s\,!}\right]$$

and μ is the mean corresponding to the p.d.f. of T_r (r>1).

It is easy to see that $\mu = \frac{1+m\bar{h}}{m}$; $\bar{h} = (1-\theta)h_1 + \theta h_2$

The p.d.f. of the waiting time of the (i + j)th conception since t_1, given that among the preceding (i + j - 1) conceptions i are of type I and (j - 1) are of type II, can easily be evaluated by putting i and (j - 1) respectively in place of (i -1) and j in equation (2). It is easy to see that

$$\begin{aligned} f_{i+j}(t/i, j-1) &= 0 ; && 0 < t \le ih_1 + (j-1)h_2 \\ &= \pi_{2,1}(t, i, j) ; && ih_1 + (j-1)h_2 < t \le (i+1)h_1 + (j-1)h_2 \\ &= \pi_{2,1}(t, i, j) - \pi_{2,2}(t, i, j) ; && (i+1)h_1 + (j-1)h_2 < t \le ih_1 + jh_2 \\ &= \pi_{2,1}(t, i, j) - \pi_{2,2}(t, i, j) - \pi_{2,3}(t, i, j) ; && t > ih_1 + jh_2 \end{aligned} \tag{3}$$

where

$$\pi_{2,1}(t, i, j) = \frac{1}{\mu}\left[1 - e^{-m(t - ih_1 - \overline{j-1}h_2)} \sum_{s=0}^{i+j-2} \frac{m^s\left(t - ih_1 - \overline{j-1}h_2\right)^s}{s\,!}\right]$$

$$\pi_{2,2}(t, i, j) = \frac{(1-\theta)}{\mu}\left[1 - e^{-m(t - \overline{i+1}h_1 - \overline{j-1}h_2)} \sum_{s=0}^{i+j-1} \frac{m^s\left(t - \overline{i+1}h_1 - \overline{j-1}h_2\right)^s}{s\,!}\right]$$

and

$$\pi_{2,3}(t, i, j) = \frac{\theta}{\mu}\left[1 - e^{-m(t - ih_1 - jh_2)} \sum_{s=0}^{i+j-1} \frac{m^s(t - ih_1 - jh_2)^s}{s\,!}\right]$$

Let us consider the probability of occurrence of the event [$X_1(\tau) = i$, $X_2(\tau) = j$], i.e. the probability of occurrence of i and j conceptions of types I and II respectively to a female during (t_1, $t_1 + \tau$) denoted by $P^*_{i,j}(\tau)$. Under the assumptions 1 to 5.

$$\begin{aligned} P^*_{0,0}(\tau) &= 1 - \int_0^\tau f_1(t)dt \\ &= 1 - \frac{\tau}{\mu} ; && 0 < \tau \le h_1 \\ &= 1 - \frac{(1-\theta)h_1}{\mu} - \frac{\theta\tau}{\mu} - \frac{(1-\theta)}{m\mu}\left(1 - e^{-m(\tau - h_1)}\right) ; && h_1 < \tau \le h_2 \\ &= \frac{(1-\theta)}{m\mu} e^{-m(\tau - h_1)} + \frac{\theta}{m\mu} e^{-m(\tau - h_2)} ; && \tau \ge h_2 \end{aligned} \tag{4}$$

which is same as

$$\begin{aligned} P^*_{0,0}(\tau) &= 1 - \phi_{1,1}(\tau, 1, 0) ; && 0 < \tau \le h_1 \\ &= 1 - \phi_{1,1}(\tau, 1, 0) + \phi_{1,4}(\tau, 1, 0) ; && h_1 < \tau \le h_2 \\ &= 1 - \phi_{1,1}(\tau, 1, 0) + \phi_{1,4}(\tau, 1, 0) + \phi_{1,5}(\tau, 1, 0) ; && \tau \ge h_2 \end{aligned} \tag{5}$$

Further, let us denote by ${}_1P_{i,j}(\tau)$ the probability of occurrence of the event [$X_1(\tau) = i$, $X_2(\tau) = j$], when $(i+j)^{th}$ conception is of type I, and by ${}_2P_{i,j}(\tau)$ the probability of the occurrence of the event $[X_1(\tau) = i, X_2(\tau) = j)]$, when $(i + j)^{th}$ conception is of type II.

It is obvious that if $(i + j)^{th}$ conception during $(t_1, t_1 + \tau)$ is of type I, then before the last conception there would be (i - 1) and j conceptions in regard to types I and II respectively.

Hence, the probability ${}_1P_{i,j}(\tau)$ is given by

$${}_1P_{i,j}(\tau)= \frac{(i+j-1)!}{(i-1)!j!}(1-\theta)^i\theta^j \int_0^\tau f_{i+j}(t/i-1,j)$$

$$\{\text{Probability of no conception during } (t_1 + t, t_1 + \tau)\}\, dt \qquad (6)$$

$$= 0 \qquad ; \quad 0 < \tau \le (i-1)h_1 + jh_2$$

$$= \frac{(i+j-1)!}{(i-1)!j!}(1-\theta)^i\theta^j \int_{(i-1)h_1+jh_2}^{\tau} \pi_{1,1}(t,i,j)\, dt$$

$$; (i-1)h_1 + jh_2 < \tau \le ih_1 + jh_2$$

$$= \frac{(i+j-1)!}{(i-1)!j!}(1-\theta)^i\theta^j \Big[\int_{(i-1)h_1+jh_2}^{\tau-h_1} \pi_{1,1}(t,i,j)\, e^{-m(\tau-t-h_1)}dt + \int_{\tau-h_1}^{\tau} \pi_{1,1}(t,i,j)dt - \int_{ih_1+jh_2}^{\tau} \pi_{1,2}(t,i,j)dt\Big]$$

$$; ih_1 + jh_2 < \tau \le (i-1)h_1 + (j+1)h_2$$

$$= \frac{(i+j-1)!}{(i-1)!j!}(1-\theta)^i\theta^j \Big[\int_{(i-1)h_1+jh_2}^{\tau-h_1} \pi_{1,1}(t,i,j)\, e^{-m(\tau-t-h_1)}dt \int_{\tau-h_1}^{\tau} \pi_{1,1}(t,i,j)dt - \int_{ih_1+jh_2}^{\tau} \pi_{1,2}(t,i,j)dt - \int_{(i-1)h_1+(j+1)h_2}^{\tau} \pi_{1,3}(t,i,j)dt\Big]$$

$$; (i-1)h_1 + (j+1)h_2 < \tau \le (i+1)h_1 + jh_2$$

$$= \frac{(i+j-1)!}{(i-1)!j!}(1-\theta)^i\theta^j \Big[\int_{(i-1)h_1+jh_2}^{\tau-h_1} \pi_{1,1}(t,i,j)\, e^{-m(\tau-t-h_1)}dt - \int_{ih_1+jh_2}^{\tau-h_1} \pi_{1,2}(t,i,j)\, e^{-m(\tau-t-h_1)}dt + \int_{\tau-h_1}^{\tau} \pi_{1,1}(t,i,j)dt - \int_{\tau-h_1}^{\tau} \pi_{1,2}(t,i,j)dt - \int_{(i-1)h_1+(j+1)h_2}^{\tau} \pi_{1,3}(t,i,j)dt\Big] ;$$

$$(i+1)h_1 + jh_2 < \tau \le ih_1 + (j+1)h_2$$

$$= \frac{(i+j-1)!}{(i-1)!j!}(1-\theta)^i\theta^j \Big[\int_{(i-1)h_1+jh_2}^{\tau-h_1} \pi_{1,1}(t,i,j)\, e^{-m(\tau-t-h_1)}dt - \int_{ih_1+jh_2}^{\tau-h_1} \pi_{1,2}(t,i,j)\, e^{-m(\tau-t-h_1)}dt -$$

$$\int_{(i-1)h_1+(j+1)h_2}^{\tau-h_1} \pi_{1,3}(t,i,j)\ e^{-m(\tau-t-h_1)}\,dt + \int_{\tau-h_1}^{\tau}\left\{\pi_{1,1}(t,i,j)-\pi_{1,2}(t,i,j)-\pi_{1,3}(t,i,j)\right\}dt\Big]$$

$$;\tau > ih_1+(j+1)h_2 \qquad (7)$$

Further, when (i + j)th conception during (t_1, $t_1 + \tau$) is of type II, then before the last conception there would be i and (j-1) conceptions that are of types I and II respectively.

Hence,

$${}_2P_{i,j}(\tau) = \frac{(i+j-1)!}{i!(j-1)!}(1-\theta)^i\theta^j \int_0^\tau f_{i+j}(t/i,j-1)$$

$$[\text{Probability of no conception during } (t_1+t,\ t_1+\tau)]\ dt \qquad (8)$$

$$= 0 \qquad ;\quad 0 < \tau \le ih_1 + (j-1)h_2$$

$$= \frac{(i+j-1)!}{i!(j-1)!}(1-\theta)^i\theta^j \int_{ih_1+(j-1)h_2}^{\tau} \pi_{2,1}(t,i,j)\,dt$$

$$;\quad ih_1+(j-1)h_2 < \tau \le (i+1)h_1+(j-1)h_2$$

$$= \frac{(i+j-1)!}{i!(j-1)!}(1-\theta)^i\theta^j \left[\int_{ih_1+(j-1)h_2}^{\tau} \pi_{2,1}(t,i,j)\,dt - \int_{(i+1)h_1+(j-1)h_2}^{\tau} \pi_{2,2}(t,i,j)\,dt\right]$$

$$;(i+1)h_1+(j-1)h_2 < \tau \le ih_1+jh_2$$

$$= \frac{(i+j-1)!}{i!(j-1)!}(1-\theta)^i\theta^j \left[\int_{ih_1+(j-1)h_2}^{\tau-h_2} \pi_{2,1}(t,i,j)\,e^{-m(\tau-t-h_2)}\,dt + \int_{\tau-h_2}^{\tau}\pi_{2,1}(t,i,j)dt - \int_{(i+1)h_1+(j-1)h_2}^{\tau} \pi_{2,2}(t,i,j)\,dt - \int_{ih_1+jh_2}^{\tau}\pi_{2,3}(t,i,j)\,dt\right]$$

$$;ih_1+jh_2 < \tau \le (i+1)h_1+jh_2$$

$$= \frac{(i+j-1)!}{i!(j-1)!}(1-\theta)^i\theta^j \left[\int_{ih_1+(j-1)h_2}^{\tau-h_2} \pi_{2,1}(t,i,j)\,e^{-m(\tau-t-h_2)}\,dt - \int_{(i+1)h_1+(j-1)h_2}^{\tau-h_2} \pi_{2,2}(t,i,j)\,e^{-m(\tau-t-h_2)}\,dt + \int_{\tau-h_2}^{\tau}\pi_{2,1}(t,i,j)dt - \int_{\tau-h_2}^{\tau}\pi_{2,2}(t,i,j)\,dt - \int_{ih_1+jh_2}^{\tau}\pi_{2,3}(t,i,j)\,dt\right]$$

$$; (i+1)h_1 + jh_2 < \tau \le ih_1 + (j+1)h_2$$

$$= \frac{(i+j-1)!}{i!(j-1)!}(1-\theta)^i\theta^j \Big[\int_{ih_1+(j-1)h_2}^{\tau-h_2} \pi_{2,1}(t,i,j)\, e^{-m(\tau-t-h_2)}\, dt - \int_{(i+1)h_1+(j-1)h_2}^{\tau-h_2} \pi_{2,2}(t,i,j)\, e^{-m(\tau-t-h_2)}\, dt - \int_{ih_1+jh_2}^{\tau-h_2} \pi_{2,3}(t,i,j)\, e^{-m(\tau-t-h_2)}\, dt + \int_{\tau-h_2}^{\tau} \{\pi_{2,1}(t,i,j) - \pi_{2,2}(t,i,j) - \pi_{2,3}(t,i,j)\}\, dt\Big]$$

$$; \tau > ih_1 + (j+1)h_2 \qquad (9)$$

Thus, under the assumptions 1 to 5, $P^*_{i,j}(\tau)$ is given by

$$P^*_{i,j}(\tau) = {}_1P_{i,j}(\tau) + {}_2P_{i,j}(\tau)$$

$$= 0 \quad ; \quad 0 < \tau \le ih_1 + (j-1)h_2$$

$$= \frac{(i+j-1)!}{i!\,j!}(1-\theta)^i\theta^j \left[j\,\emptyset_{2,1}(\tau,i,j)\right] ; \quad ih_1 + (j-1)h_2 < \tau \le (i-1)h_1 + jh_2$$

$$= \frac{(i+j-1)!}{i!\,j!}(1-\theta)^i\theta^j \left[i\,\emptyset_{1,1}(\tau,i,j) + j\,\emptyset_{2,1}(\tau,i,j)\right]$$

$$(i-1)h_1 + jh_2 < \tau \le (i+1)h_1 + (j-1)h_2$$

$$= \frac{(i+j-1)!}{i!\,j!}(1-\theta)^i\theta^j \left[i\,\emptyset_{1,1}(\tau,i,j) + j\{\emptyset_{2,1}(\tau,i,j) - \emptyset_{2,2}(\tau,i,j)\}\right]$$

$$; (i+1)h_1 + (j-1)h_2 < \tau \le ih_1 + jh_2$$

$$= \frac{(i+j-1)!}{i!\,j!}(1-\theta)^i\theta^j \Big[i\{\emptyset_{1,2}(\tau,i,j) + \emptyset_{1,3}(\tau,i,j) - \emptyset_{1,4}(\tau,i,j)\} + j\{-\emptyset_{2,2}(\tau,i,j) + \emptyset_{2,3}(\tau,i,j) + \emptyset_{2,4}(\tau,i,j) - \emptyset_{2,5}(\tau,i,j)\}\Big]$$

$$; ih_1 + jh_2 < \tau \le (i-1)h_1 + (j+1)h_2$$

$$= \frac{(i+j-1)!}{i!\,j!}(1-\theta)^i\theta^j \Big[i\{\emptyset_{1,2}(\tau,i,j) + \emptyset_{1,3}(\tau,i,j) - \emptyset_{1,4}(\tau,i,j) - \emptyset_{1,5}(\tau,i,j)\} + j\{-\emptyset_{2,2}(\tau,i,j) + \emptyset_{2,3}(\tau,i,j) + \emptyset_{2,4}(\tau,i,j) - \emptyset_{2,5}(\tau,i,j)\}\Big]$$

$$; (i-1)h_1 + (j+1)h_2 < \tau \le (i+1)h_1 + jh_2$$

$$= \frac{(i+j-1)!}{i!\,j!}(1-\theta)^i\theta^j \Big[i\{\emptyset_{1,2}(\tau,i,j) + \emptyset_{1,3}(\tau,i,j) - \emptyset_{1,5}(\tau,i,j) - \emptyset_{1,6}(\tau,i,j) - \emptyset_{1,7}(\tau,i,j)\} + j\{\emptyset_{2,3}(\tau,i,j) +$$

$\emptyset_{2,4}(\tau,\mathrm{i},\mathrm{j}) - \emptyset_{2,5}(\tau,\mathrm{i},\mathrm{j}) - \emptyset_{2,6}(\tau,\mathrm{i},\mathrm{j}) - \emptyset_{2,7}(\tau,\mathrm{i},\mathrm{j})\}\big]$

$$; (\mathrm{i}+1)\mathrm{h}_1 + \mathrm{jh}_2 < \tau \le \mathrm{ih}_1 + (\mathrm{j}+1)\mathrm{h}_2$$

$$= \frac{(\mathrm{i}+\mathrm{j}-1)!}{\mathrm{i}!\,\mathrm{j}!}(1-\theta)^{\mathrm{i}}\theta^{\mathrm{j}}\Big[\mathrm{i}\,\{\emptyset_{1,2}(\tau,\mathrm{i},\mathrm{j}) + \emptyset_{1,3}(\tau,\mathrm{i},\mathrm{j}) - \emptyset_{1,6}(\tau,\mathrm{i},\mathrm{j}) - \emptyset_{1,7}(\tau,\mathrm{i},\mathrm{j}) - \emptyset_{1,8}(\tau,\mathrm{i},\mathrm{j}) - \emptyset_{1,9}(\tau,\mathrm{i},\mathrm{j})\} + \mathrm{j}\,\{\emptyset_{2,3}(\tau,\mathrm{i},\mathrm{j}) + \emptyset_{2,4}(\tau,\mathrm{i},\mathrm{j}) - \emptyset_{2,6}(\tau,\mathrm{i},\mathrm{j}) - \emptyset_{2,7}(\tau,\mathrm{i},\mathrm{j}) - \emptyset_{2,8}(\tau,\mathrm{i},\mathrm{j}) - \emptyset_{2,9}(\tau,\mathrm{i},\mathrm{j})\}\Big]$$

$$; \ \tau \ge \mathrm{ih}_1 + (\mathrm{j}+1)\mathrm{h}_2$$

$$\mathrm{i} = 0, 1, 2, \ldots, \mathrm{n}_1; \ \mathrm{j} = 0, 1, 2, \ldots, \mathrm{n}_2 \ \text{and} \ (\mathrm{i}+\mathrm{j}) \neq 0 \qquad (10)$$

where $\phi_{k,l}$ (τ, i, j)'s (k = 1, 2, ; l = 1, 2, …, 9) are obtained by the following expressions after replacing T by τ .

$$\phi_{1,1}(\mathrm{T},\mathrm{i},\mathrm{j}) = \int_{(\mathrm{i}-1)\mathrm{h}_1+\mathrm{jh}_2}^{\mathrm{T}} \pi_{1,1}(\mathrm{t},\mathrm{i},\mathrm{j})\mathrm{dt}$$

$$\phi_{1,2}(\mathrm{T},\mathrm{i},\mathrm{j}) = \int_{(\mathrm{i}-1)\mathrm{h}_1+\mathrm{jh}_2}^{\mathrm{T}-\mathrm{h}_1} \pi_{1,1}(\mathrm{t},\mathrm{i},\mathrm{j})\ \mathrm{e}^{-\mathrm{m}\,(\mathrm{T}-\mathrm{t}-\mathrm{h}_1)}\mathrm{dt}$$

$$\phi_{1,3}(\mathrm{T},\mathrm{i},\mathrm{j}) = \int_{\mathrm{T}=\mathrm{h}_1}^{\mathrm{T}} \pi_{1,1}(\mathrm{t},\mathrm{i},\mathrm{j})\mathrm{dt}$$

$$\phi_{1,4}(\mathrm{T},\mathrm{i},\mathrm{j}) = \int_{\mathrm{ih}_1+\mathrm{jh}_2}^{\mathrm{T}} \pi_{1,2}(\mathrm{t},\mathrm{i},\mathrm{j})\mathrm{dt}$$

$$\phi_{1,5}(\mathrm{T},\mathrm{i},\mathrm{j}) = \int_{(i-1)\mathrm{h}_1+(\mathrm{j}+1)\mathrm{h}_2}^{\mathrm{T}} \pi_{1,3}(\mathrm{t},\mathrm{i},\mathrm{j})\mathrm{dt}$$

$$\phi_{1,6}(\mathrm{T},\mathrm{i},\mathrm{j}) = \int_{i\mathrm{h}_1+\mathrm{jh}_2}^{\mathrm{T}-\mathrm{h}_1} \pi_{1,2}(\mathrm{t},\mathrm{i},\mathrm{j})\ \mathrm{e}^{-\mathrm{m}\,(\mathrm{T}-\mathrm{t}-\mathrm{h}_1)}\ \mathrm{dt}$$

$$\phi_{1,7}(\mathrm{T},\mathrm{i},\mathrm{j}) = \int_{\mathrm{T}-\mathrm{h}_1}^{\mathrm{T}} \pi_{1,2}(\mathrm{t},\mathrm{i},\mathrm{j})\ \mathrm{dt}$$

$$\phi_{1,8}(\mathrm{T},\mathrm{i},\mathrm{j}) = \int_{(i-1)\mathrm{h}_1+(\mathrm{j}+1)\mathrm{h}_2}^{\mathrm{T}-\mathrm{h}_1} \pi_{1,3}(\mathrm{t},\mathrm{i},\mathrm{j})\ \mathrm{e}^{-\mathrm{m}\,(\mathrm{T}-\mathrm{t}-\mathrm{h}_1)}\ \mathrm{dt}$$

$$\phi_{1,9}(\mathrm{T},\mathrm{i},\mathrm{j}) = \int_{\mathrm{T}-\mathrm{h}_1}^{\mathrm{T}} \pi_{1,3}(\mathrm{t},\mathrm{i},\mathrm{j})\ \mathrm{dt}$$

$$\phi_{2,1}(\mathrm{T},\mathrm{i},\mathrm{j}) = \int_{\mathrm{ih}_1+(\mathrm{j}-1)\mathrm{h}_2}^{\mathrm{T}} \pi_{2,1}(\mathrm{t},\mathrm{i},\mathrm{j})\ \mathrm{dt}$$

$$\phi_{2,2}(\mathrm{T},\mathrm{i},\mathrm{j}) = \int_{(\mathrm{i}+1)\mathrm{h}_1+(\mathrm{j}-1)\mathrm{h}_2}^{\mathrm{T}} \pi_{2,2}(\mathrm{t},\mathrm{i},\mathrm{j})\ \mathrm{dt}$$

$$\phi_{2,3}(T,i,j) = \int_{ih_1+(j-1)h_2}^{T} \pi_{2,1}(t,i,j)\ e^{-m(T-t-h_2)}\,dt$$

$$\phi_{2,4}(T,i,j) = \int_{T-h_2}^{T} \pi_{2,1}(t,i,j)\,dt$$

$$\phi_{2,5}(T,i,j) = \int_{ih_1+jh_2}^{T} \pi_{2,3}(t,i,j)\,dt$$

$$\phi_{2,6}(T,i,j) = \int_{(i+1)h_1+(j-1)h_2}^{T-h_2} \pi_{2,2}(t,i,j)\ e^{-m(T-t-h_2)}\,dt$$

$$\phi_{2,7}(T,i,j) = \int_{T-h_2}^{T} \pi_{2,2}(t,i,j)\,dt$$

$$\phi_{2,8}(T,i,j) = \int_{ih_1+jh_2}^{T-h_2} \pi_{2,3}(t,i,j)\,e^{-m(T-t-h_2)}\,dt$$

$$\phi_{2,9}(T,i,j) = \int_{T-h_2}^{T} \pi_{2,3}(t,i,j)\,dt$$

Now, the probability of i and j conceptions of types I and II respectively during (t_1, t_2), given that the female is fecund at t_1 and remains so throughout the period of observation, is $P^*_{1,j}(T)$, which is obtained by substituting T for τ in $P^*_{1,j}(\tau)$. Further the probability of i and j conceptions of types I and II respectively to a female during (t_1, t_2), given that she was fecund at t_1 and becomes sterile during $(\tau, \tau + d\tau)$, is given by

$$\frac{1}{T} P^*_{1,j}(\tau)\,d\tau \quad , \quad 0 < \tau \leq T.$$

Now, a_1a_2 is the probability that a female fecund at the start of the observational period, remains so throughout the period; while $a_1(1\text{-}a_2)$ is the probability that a female fecund at the start of the observational period becomes sterile at any point during the period. Thus, the joint probability function of $X_1(T)$ and $X_2(T)$, under assumptions 1 to 7, is given by

$$P_{0,0}(T) = (1-a_1) + a_1\,a_2\,P^*_{0,0}(T) + \frac{a_1(1-a_2)}{T}\int_0^T P^*_{0,0}(\tau)\,d\tau \ ; \tag{11}$$

$$= (1-a_1) + a_1\,a_2\left[1 - \phi_{1,1}(T,1,0)\right] + \frac{a_1(1-a_2)}{T}\left[T - \psi_{1,1}(T,1,0)\right];$$

$$0 < T \leq h_1$$

$$= (1-a_1) + a_1\,a_2\left[1 - \phi_{1,1}(T,1,0) + \phi_{1,4}(T,1,0)\right] + \frac{a_1(1-a_2)}{T}\left[T - \psi_{1,1}(T,1,0) + \psi_{1,4}(T,1,0)\right]$$

$$; h_1 < T \leq h_2$$

$$= (1-a_1) + a_1\, a_2 \left[1 - \phi_{1,1}\,(T,1,0) + \phi_{1,4}\,(T,1,0) + \phi_{1,5}\,(T,1,0)\right] + \frac{a_1(1-a_2)}{T}\left[T - \psi_{1,1}(T,1,0) + \psi_{1,4}(T,1,0) + \psi_{1,5}(T,1,0)\right]$$

$$; T \geq h_2 \quad (12)$$

$$P_{i,j}\,(T) = a_1\, a_2\, P^*_{0,0}\,(T) + \frac{a_1(1-a_2)}{T}\int_0^T P^*_{1,j}\,(\tau)\, d\tau \qquad (13)$$

$$P_{i,j}\,(T) = 0 \qquad ;\ 0 < T \leq ih_1 + (j-1)h_2$$

$$P_{i,j}\,(T) = a_1\, a_2 \frac{(i+j-1)!}{i\,!\; j\,!}(1-\theta)^i\ \theta^j\ j\ \phi_{2,1}\,(T,i,j)$$

$$+\frac{a_1(1-a_2)}{T}\frac{(i+j-1)!}{i\,!\,j\,!}(1-\theta)^i\ \theta^j\ j\,\psi_{2,1}(T,i,j)$$

$$;\ ih_1 + (j-1)h_2 < T \leq (i-1)h_1 + jh_2$$

$$= a_1\, a_2 \frac{(i+j-1)!}{i\,!\,j\,!}(1-\theta)^i\ \theta^j \left[i\ \phi_{1,1}\,(T,i,j) + j\ \phi_{2,1}\,(T,i,j)\right]$$

$$+\frac{a_1(1-a_2)}{T}\frac{(i+j-1)!}{i\,!\,j\,!}(1-\theta)^i\ \theta^j \left[i\,\psi_{1,1}(T,i,j) + j\,\psi_{2,1}(T,i,j)\right]$$

$$;\ (i-1)h_1 + jh_2 < T \leq (i+1)h_1 + (j-1)h_2$$

$$= a_1\, a_2 \frac{(i+j-1)!}{i\,!\,j\,!}(1-\theta)^i\ \theta^j \left[i\left\{\phi_{1,1}\,(T,i,j)\right\} + j\left\{\phi_{2,1}\,(T,i,j) - \phi_{2,2}\,(T,i,j)\right\}\right] + \frac{a_1(1-a_2)}{T}\frac{(i+j-1)!}{i\,!\,j\,!}(1-\theta)^i\ \theta^j \left[i\ \left\{\psi_{1,1}(T,i,j)\right\} + j\left\{\psi_{2,1}(T,i,j) - \psi_{2,2}(T,i,j)\right\}\right]$$

$$;\ (i+1)h_1 + (j-1)h_2 < T \leq ih_1 + jh_2$$

$$= a_1\, a_2 \frac{(i+j-1)!}{i\,!\,j\,!}(1-\theta)^i\ \theta^j \left[i\left\{\phi_{1,2}\,(T,i,j) + \phi_{1,3}\,(T,i,j) - \phi_{1,4}\,(T,i,j)\right\} + j\left\{\phi_{2,3}\,(T,i,j) + \phi_{2,4}\,(T,i,j) - \phi_{2,2}\,(T,i,j) - \phi_{2,5}\,(T,i,j)\right\}\right] + \frac{a_1(1-a_2)}{T}\frac{(i+j-1)!}{i\,!\,j\,!}(1-\theta)^i\ \theta^j \left[i\ \left\{\psi_{1,1(1)}(T,i,j) + \psi_{1,2}(T,i,j) + \psi_{1,3}(T,i,j) - \psi_{1,4}(T,i,j)\right\} + j\left\{\psi_{2,1(1)}(T,i,j) - \psi_{2,2}(T,i,j) + \psi_{2,3}(T,i,j) + \psi_{2,4}(T,i,j) - \psi_{2,5}(T,i,j)\right\}\right]$$

$$; \ ih_1 + jh_2 < T \leq (i-1)h_1 + (j+1)h_2$$

$= a_1 a_2 \frac{(i+j-1)!}{i!\,j!}(1-\theta)^i \theta^j \Big[i \big\{ \phi_{1,2}(T,i,j) + \phi_{1,3}(T,i,j) - \phi_{1,4}(T,i,j) - \phi_{1,5}(T,i,j) \big\} + j \big\{ -\phi_{2,2}(T,i,j) + \phi_{2,3}(T,i,j) + \phi_{2,4}(T,i,j) - \phi_{2,5}(T,i,j) \big\} \Big] + \frac{a_1(1-a_2)}{T} \frac{(i+j-1)!}{i!\,j!}(1-\theta)^i \theta^j \Big[i \big\{ \psi_{1,1(1)}(T,i,j) + \psi_{1,2}(T,i,j) + \psi_{1,3}(T,i,j) - \psi_{1,4}(T,i,j) - \psi_{1,5}(T,i,j) \big\} + j \big\{ \psi_{2,1(1)}(T,i,j) - \psi_{2,2}(T,i,j) + \psi_{2,3}(T,i,j) + \psi_{2,4}(T,i,j) - \psi_{2,5}(T,i,j) \big\} \Big]$

$$; \ (i-1)h_1 + (j+1)h_2 < T \leq (i+1)h_1 + jh_2$$

$$= a_1 a_2 \frac{(i+j-1)!}{i!\,j!}(1-\theta)^i \theta^j \Big[i \big\{ \phi_{1,2}(T,i,j) + \phi_{1,3}(T,i,j) - \phi_{1,5}(T,i,j) - \phi_{1,6}(T,i,j) - \phi_{1,7}(T,i,j) \big\} + j \big\{ \phi_{2,3}(T,i,j) + \phi_{2,4}(T,i,j) - \phi_{2,5}(T,i,j) - \phi_{2,6}(T,i,j) - \phi_{2,7}(T,i,j) \big\} \Big]$$

$+ \frac{a_1(1-a_2)}{T} \frac{(i+j-1)!}{i!\,j!}(1-\theta)^i \theta^j \Big[i \big\{ \psi_{1,1(1)}(T,i,j) + \psi_{1,2}(T,i,j) + \psi_{1,3}(T,i,j) - \psi_{1,4(1)}(T,i,j) - \psi_{1,5}(T,i,j) - \psi_{1,6}(T,i,j) - \psi_{1,7}(T,i,j) \big\} + j \big\{ \psi_{2,1(1)}(T,i,j) - \psi_{2,2(1)}(T,i,j) + \psi_{2,3}(T,i,j) + \psi_{2,4}(T,i,j) - \psi_{2,5}(T,i,j) - \psi_{2,6}(T,i,j) - \psi_{2,7}(T,i,j) \big\} \Big]$

$$; \ (i+1)h_1 + jh_2 < T \leq ih_1 + (j+1)h_2$$

$= a_1 a_2 \frac{(i+j-1)!}{i!\,j!}(1-\theta)^i \theta^j [\, i \{\phi_{1,2}(T,i,j) + \phi_{1,3}(T,i,j) - \phi_{1,6}(T,i,j) - \phi_{1,7}(T,i,j) - \phi_{1,8}(T,i,j) - \phi_{1,9}(T,i,j)\} + j \{\phi_{2,3}(T,i,j) + \phi_{2,4}(T,i,j) - \phi_{2,6}(T,i,j) - \phi_{2,7}(T,i,j) - \phi_{2,8}(T,i,j) - \phi_{2,9}(T,i,j)\}]$

$+ \frac{a_1(1-a_2)}{T} \frac{(i+j-1)!}{i!\,j!}(1-\theta)^i \theta^j [\, i \{\psi_{1,1(1)}(T,i,j) + \psi_{1,2}(T,i,j) + \psi_{1,3}(T,i,j) - \psi_{1,4(1)}(T,i,j) - \psi_{1,5(1)}(T,i,j) - \psi_{1,6}(T,i,j) - \psi_{1,7}(T,i,j) - \psi_{1,8}(T,i,j) - \psi_{1,9}(T,i,j)\} + j \{\psi_{2,1(1)}(T,i,j) - \psi_{2,2(1)}(T,i,j) + \psi_{2,3}(T,i,j) + \psi_{2,4}(T,i,j) - \psi_{2,5(1)}(T,i,j) - \psi_{2,6}(T,i,j) - \psi_{2,7}(T,i,j) - \psi_{2,8}(T,i,j) - \psi_{2,9}(T,i,j)\}]$

$; \ T \geq ih_1 + (j+1)h_2$

for $i = 0, 1, 2, \ldots, n_1$; $j = 0, 1, 2, \ldots, n_2$ and $i + j \neq 0$ (14)

where $\psi_{k,l}(T, i, j)$'s ($k = 1, 2,; \; l = 1, 2, \ldots, 9$) are given by

$$\psi_{1,1}(T,i,j) = \int_{(i-1)h_1+jh_2}^{T} \phi_{1,1}(\tau,i,j)\,d\tau$$

$$\psi_{1,1\,(1)}(T,i,j) = \int_{(i-1)h_1+jh_2}^{ih_1+jh_2} \phi_{1,1}(\tau,i,j)\,d\tau$$

$$\psi_{1,2}(T,i,j) = \int_{ih_1+jh_2}^{T} \phi_{1,2}(\tau,i,j)\,d\tau$$

$$\psi_{1,3}(T,i,j) = \int_{ih_1+jh_2}^{T} \phi_{1,3}(\tau,i,j)\,d\tau$$

$$\psi_{1,4}(T,i,j) = \int_{ih_1+jh_2}^{T} \phi_{1,4}(\tau,i,j)\,d\tau$$

$$\psi_{1,4\,(1)}(T,i,j) = \int_{ih_1+jh_2}^{(i+1)h_1+jh_2} \phi_{1,4}(\tau,i,j)\,d\tau$$

$$\psi_{1,5}(T,i,j) = \int_{(i-1)h_1+(j+1)h_2}^{T} \phi_{1,5}(\tau,i,j)\,d\tau$$

$$\psi_{1,5\,(1)}(T,i,j) = \int_{(i-1)h_1+(j+1)h_2}^{ih_1+(j+1)h_2} \phi_{1,5}(\tau,i,j)\,d\tau$$

$$\psi_{1,6}(T,i,j) = \int_{(i+1)h_1+jh_2}^{T} \phi_{1,6}(\tau,i,j)\,d\tau$$

$$\psi_{1,7}(T,i,j) = \int_{(i+1)h_1+jh_2}^{T} \phi_{1,7}(\tau,i,j)\,d\tau$$

$$\psi_{1,8}(T,i,j) = \int_{ih_1+(j+1)h_2}^{T} \phi_{1,8}(\tau,i,j)\,d\tau$$

$$\psi_{1,9}(T,i,j) = \int_{ih_1+(j+1)h_2}^{T} \phi_{1,9}(\tau,i,j)\,d\tau$$

$$\psi_{2,1}(T,i,j) = \int_{ih_1+(j-1)h_2}^{T} \phi_{2,1}(\tau,i,j)\,d\tau$$

$$\psi_{2,1\,(1)}(T,i,j) = \int_{ih_1+(j-1)h_2}^{ih_1+jh_2} \phi_{2,1}(\tau,i,j)\,d\tau$$

$$\psi_{2,2}(T,i,j) = \int_{(i+1)h_1+(j-1)h_2}^{T} \phi_{2,2}(\tau,i,j)\,d\tau$$

$$\psi_{2,2\,(1)}(T,i,j) = \int_{(i+1)h_1+(j-1)h_2}^{(i+1)h_1+jh_2} \phi_{2,2}(\tau,i,j)\,d\tau$$

$$\psi_{2,3}(T,i,j) = \int_{ih_1+jh_2}^{T} \phi_{2,3}(\tau,i,j)\,d\tau$$

$$\psi_{2,4}(\mathrm{T},\mathrm{i},\mathrm{j}) = \int_{\mathrm{ih}_1+\mathrm{jh}_2}^{\mathrm{T}} \phi_{2,4}(\tau,\mathrm{i},\mathrm{j})\, d\tau$$

$$\psi_{2,5}(\mathrm{T},\mathrm{i},\mathrm{j}) = \int_{\mathrm{ih}_1+\mathrm{jh}_2}^{\mathrm{T}} \phi_{2,5}(\tau,\mathrm{i},\mathrm{j})\, d\tau$$

$$\psi_{2,5\,(1)}(\mathrm{T},\mathrm{i},\mathrm{j}) = \int_{\mathrm{ih}_1+\mathrm{jh}_2}^{\mathrm{ih}_1+(\mathrm{j}+1)\mathrm{h}_2} \phi_{2,5}(\tau,\mathrm{i},\mathrm{j})\, d\tau$$

$$\psi_{2,6}(\mathrm{T},\mathrm{i},\mathrm{j}) = \int_{(\mathrm{i}+1)\mathrm{ih}_1+\mathrm{jh}_2}^{\mathrm{T}} \phi_{2,6}(\tau,\mathrm{i},\mathrm{j})\, d\tau$$

$$\psi_{2,7}(\mathrm{T},\mathrm{i},\mathrm{j}) = \int_{(\mathrm{i}+1)\mathrm{ih}_1+\mathrm{jh}_2}^{\mathrm{T}} \phi_{2,7}(\tau,\mathrm{i},\mathrm{j})\, d\tau$$

$$\psi_{2,8}(\mathrm{T},\mathrm{i},\mathrm{j}) = \int_{\mathrm{ih}_1+(\mathrm{j}+1)\mathrm{h}_2}^{\mathrm{T}} \phi_{2,8}(\tau,\mathrm{i},\mathrm{j})\, d\tau$$

$$\psi_{2,9}(\mathrm{T},\mathrm{i},\mathrm{j}) = \int_{\mathrm{ih}_1+(\mathrm{j}+1)\mathrm{h}_2}^{\mathrm{T}} \phi_{2,9}(\tau,\mathrm{i},\mathrm{j})\, d\tau$$

Application

To illustrate the application of the model an observed distribution has been taken from the survey "Rural Development and Population Growth (1978)". Table given presents the distribution of number of children within a six-year period, to females aged 40-45 on the reference date of the survey. The table has been prepared by utilizing the information on the number of births during the seven years antecedent to the reference date. The births have been classified under two heads : the children dying within a year, and those surviving for at least one year after birth. The births which took place during the last one year failed to fulfill the criterion of one-year exposure period, and in whose cases the factor of infantile mortality could not be ascertained, were ignored. Thus on average the births relate to the fertility performance of a female during ages 35.5 to 41.5 instead of 35.5 to 42.5, owing to the exclusion of births during one year before the reference date. Since, the likelihood of the sterility being greater in the later phase of the reproductive period, the estimation of the parameters is done by utilizing the observed set of data related to the later part of the reproductive span. Also the females using hundred percent effective contraceptives are not taken into consideration, since we are considering only onset of natural sterility.

The present model consists of six parameters viz. $(1 - \theta)$, h_1, h_2, a_1, a_2 and m. By assuming $(1 - \theta)$, h_1 and h_2, we have estimated the parameters a_1, a_2 and m by equating the theoretical expressions of mean, variance and probability of zero birth to their respective observed values. From the survey data it is observed that the infant mortality rate is around 150 per thousand births. Hence

$(1 - \theta)$ has been taken here as 0.15. Further, for the estimation of the parameters, h_1 and h_2 have been taken as 1.00 year and 1.5 years. Here T = 6 years. The estimates $\hat{a}_1\hat{a}_2$ and $\hat{m}$ of a_1, a_2 and m are obtained as

$$\hat{a}_1 = 0.710, \qquad \hat{a}_2 = 0.649 \qquad \text{and} \quad \hat{m} = 0.401.$$

With these estimates the expected frequencies for the distribution have been computed and presented in the Table. The value of χ^2 at 3 degrees of freedom is 2.2066.

If we take $a_2 = 1$. our model reduces to that of Singh, (1978). In view of this, we have also obtained the estimates of a_1 and m by using the theoretical expressions of probability of zero birth and mean. It this case the estimates $\hat{a}_1, \hat{m}$ of a_1 and m are obtained respectively as

$$\hat{a}_1 = 0.650, \quad \text{and} \quad \hat{m} = 0.343.$$

The expected frequencies and value of χ^2, based on the above estimates, are also presented in the Table given below. Here value of χ^2 is obtained as 2.9869 at 4 degrees of freedom.

Both the values of χ^2, in the Table below, are found to be insignificant at 5 per cent level of significance. However, the present model may be more suitable because of its assumption that sterility increases with a female's age after the thirty fifth year of her life.

Conclusion and Interpretation

The proposed bivariate probability distribution is useful for interpreting the observed data in respect of the later segment of the reproductive life of a female. Such a distribution permits relatively larger degrees of freedom for the same duration of observational period.

The estimate of m obtained in the present model is 0.401, which is in agreement with the value of m obtained by Srivastava and Singh (1989) for the same group of females. Also, the proportion of fecund females (i.e., a_1) at the average age 35.5 is found to be 0.710, which is also similar to that obtained by Srivastava and Singh (1989). The pattern of estimates is the same.

When a_2 is equal to one, the estimates of a_1 and a_2, obtained here, indicate that about 16 percent females remain fecund at the end of the observational period, which is higher than that one obtained for the females of the same age group by Srivastava and Singh (1989). The higher estimates of fecund proportion may be due to the fact that it corresponds to the females of age 41.5 instead of 42.5, to which reference has already been made.

In the end it may be concluded that the estimates based on either of the two models - the univariate and the bivariate - presented by Srivastava and Singh (1989) and in this paper may be approximately equal.

Table : Observed and expected bivariate distribution of the number of live birth to Females of age 40-45 years during six years

Number of birth : Children dying within a year	**Number of birth : Children surviving more than a year**						**Total**
		0	**1**	**2**	**3**	**4**	
	0	179	99	65	16	0	359
0	A_1	179.00	99.11	65.94	12.31	0.24	355.60
	A_2	179.00	99.79	67.76	11.60	0.20	355.35
	0	11	20	3	3		37
1	A_1	15.14	22.61	8.19	0.41		46.35
	A_2	11.85	23.64	7.75	0.34		46.58
	0	3	5	0	1		9
2	A_1	1.94	1.73	0.20	0.00		3.87
	A_2	1.81	1.64	0.17	0.00		3.62
	0	0	1				1
3	A_1	0.12	0.06				0.18
	A_2	0.11	0.34				0.45
	0	193	125	68	20	0	
Total	A_1	196.20	123.51	73.33	12.72	0.24	406
	A_2	195.77	122.41	75.68	11.94	0.20	

Note : In each cell, A_1 and A_2 denote the expected frequencies according to the present model and Singh's model (1978) respectively. Calculated values of χ^2 are

: 2.2066 with 3 d.f. for the proposed model

: 2.9869 with 4 d.f. for Singh's model (1978)

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