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AN IMPROVED CLASS OF TWO PHASE SAMPLING ESTIMATORS FOR POPULATION MEAN USING AUXILIARY CHARACTER IN PRESENCE OF NON RESPONSE

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ABSTRACT

An improved class of two phase sampling estimators for population mean using auxiliary character in presence of nonresponse has been proposed under certain specified conditions. The properties of proposed estimators have been studied. An empirical study has been given in the support of problem.

Keywords : Two phase sampling, non response

INTRODUCTION

While conducting the sample surveys in the field of agriculture, social sciences and medical sciences, the problem of non response is very common in practice. The problem of estimation of population mean using the technique of sub sampling from non respondents was first introduced by Hansen and Hurwitz (1946). Further Rao (1986, 90) have suggested the use of auxiliary character with known population mean in presence of nonresponse problems to increase the efficiency of the estimator suggested by Hansen and Hurwitz (1946) and consequently the conventional and alternative ratio and regression type estimators are proposed. Further transformed product and ratio type estimators for population mean has been considered by Khare and Srivastava (1996, 97). The generalized estimator for population mean in presence of nonresponse has been considered by Khare and Srivastava (2000). In some cases, the population mean of the auxiliary character may not be known viz. while estimating the yield of a crop, the average area of the plot may not be known. In such cases two phase sampling estimators for population mean using auxiliary character in presence of nonresponse, have been proposed by Khare and Srivastava (1993, 95, 99).

In the present paper, we have proposed an improved class of two phase sampling estimator for population mean using auxiliary character in the presence of non-response. The properties of the proposed class of estimators have been studied. The values of the mean square error of the proposed class of estimators have been obtained under the optimum conditions. An empirical study has been given in support of the problem.

THE PROPOSED CLASS OF ESTIMATORS AND RELEVANT ESTIMATORS

Let $(Y_1, Y_2, ...Y_N)$ and $(X_1, X_2, ...X_N)$ be the values of N units of the population for y and x character having population mean $\bar{Y}$ and $\bar{X}$ respectively. The whole population is supposed to be divided into two parts, (i) Response group and (ii) Non-response group, having N_1 and N_2 (unknown) units in the groups respectively.

Using the Hansen- Hurwitz technique, a sample of size n is drawn from the population of size N by using SRSWOR method of sampling. It is observed that among n units, n_1 units respond and n_2 units do not respond. Further, a subsample of size $r(=\frac{n_2}{k}, k>1)$ is drawn from n_2 non-responding units. The estimator proposed by Hansen and Hurwitz (1946) is given by

$$\bar{y}^* = \frac{n_1}{n}\bar{y}_1 + \frac{n_2}{n}\bar{y}_2^{'}, \tag{2.1}$$

where $\bar{y}_1$ and $\bar{y}_2^{'}$ denote the sample means of y based on n_1 and r units respectively.

In the case when $\bar{X}$ is not known, then we device a two phase sampling plan which is given as follows:

TWO PHASE SAMPLING PLAN

From the population of size N, a larger sample of size $n^{'}$ is drawn and the auxiliary character x is observed. Let the sample mean of x for $n^{'}$ units is denoted by $\bar{x}^{'}$. Again from $n^{'}$ units selected at primary level, a sample of size $n(<n^{'})$ is drawn and it was observed that n_1 units respond and n_2 units do not respond. Further from n_2 non-responding units, a sub sample of size $r(=\frac{n_2}{k}, k>1)$ is drawn and the study character y is observed. We also denote,

$$\bar{x}^* = \frac{n_1}{n}\bar{x}_1 + \frac{n_2}{n}\bar{x}_2' \quad , \tag{2.2}$$

where $\bar{x}_1$ and $\bar{x}_2'$ are the sample means of size n_1 and r units for the auxiliary character x.

Now using $\bar{x}'$ and the value of x corresponding to incomplete information on y from the second phase sample of size n, the proposed class of two phase sampling estimators for $\bar{Y}$ is given by

$$T_1 = h(w, u_1), \tag{2.3}$$

where $$w = \bar{y}^*,$$

$$u_1 = \frac{\bar{x}^*}{\bar{x}'}, \qquad h(\bar{Y}, 1) = \bar{Y} \;\; h_1(\bar{Y}, 1) \;\; ,$$

$$h_1\left(\bar{Y}, 1\right) = \left(\frac{\partial}{\partial w} h\left(w, u_1\right)\right)_{\left(\bar{Y}, 1\right)}$$

Further, another class of two phase sampling estimators T_2 for $\bar{Y}$ is proposed by using $\bar{x}'$, incomplete information on y and complete information on x from a sample of size n which is given by

$$T_2 = h(w, u_2), \tag{2.4}$$

where $$u_2 = \frac{\bar{x}}{\bar{x}'},$$

$$h(\bar{Y}, 1) = \bar{Y} \;\; h_1(\bar{Y}, 1)$$

and $$h_1\left(\bar{Y}, 1\right) = \left(\frac{\partial}{\partial w} h\left(w, u_2\right)\right)_{\left(\bar{Y}, 1\right)}$$

The function $h(w, u_i), i = 1, 2$ is satisfying the following conditions:-

(i) Whatever be the sample chosen, w, u_i assumes positive values in a bounded subset D_i, containing the points $w = \bar{Y}$ and $u_i = 1 (i = 1, 2)$ on a real line respectively.

(ii) The function $h(w, u_i)$ and its first and second order partial derivatives with respect to w and u_i exists and are assumed to be continuous and bounded in $D_i\,(i=1,2)$ on a real line.

Here $h_1(w,u_i)$ and $h_{2(i)}(w,u_i)$ denote the first partial derivative of $h(w,u_i)$ with respect to w and $u_i\,(i=1,2)$ respectively. The second partial derivatives of $h(w,u_i)$ with respect to w and u_i is given by $h_{11}(w,u_i)$ and $h_{22(i)}(w,u_i)$ respectively. The first partial derivative of $h_{2(i)}(w,u_i)$ with respect to w is denoted by $h_{12(i)}(w,u_i)$. The conventional and alternative two phase sampling ratio (t_1,t_2) and regression type (t_3,t_4) estimators proposed by Khare and Srivastava (1993) and Khare (1992) are as follows:

$$t_1=\frac{\bar{y}^*}{\bar{x}^*}\bar{x}', \qquad t_2=\frac{\bar{y}^*}{\bar{x}}\bar{x}', \tag{2.5}$$

$$t_3=\bar{y}^*+b^*(\bar{x}'-\bar{x}^*), \quad t_4=\bar{y}^*+b^{**}(\bar{x}'-\bar{x}) \tag{2.6}$$

where $b^*=\dfrac{\hat{S}_{yx}}{\hat{S}_x^2}$, $b^{**}=\dfrac{\hat{S}_{yx}}{s_x^2}$ and $s_x^2=\dfrac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$.

The mean square error of the estimator t_1,t_2,t_3 and t_4 are given by

$$MSE(t_1)=\left(\frac{1}{n}-\frac{1}{n'}\right)S_{y.x}+\frac{f'}{n'}S_y^2+\frac{W_2(k-1)}{n}S_{y.x(2)}, \tag{2.7}$$

$$MSE(t_2)=\left(\frac{1}{n}-\frac{1}{n'}\right)S_{y.x}+\frac{f'}{n'}S_y^2+\frac{W_2(k-1)}{n}S_{y(2)}^2, \tag{2.8}$$

$$MSE(t_3)=\left(\frac{1}{n}-\frac{1}{n'}\right)S_y^2(1-\rho^2)+\frac{f'}{n'}S_y^2+\frac{W_2(k-1)}{n}\left[S_{y(2)}^2+\beta^2S_{x(2)}^2-2\beta\,S_{yx(2)}\right] \tag{2.9}$$

and $$MSE(t_4)=\left(\frac{1}{n}-\frac{1}{n'}\right)S_y^2(1-\rho^2)+\frac{f'}{n'}S_y^2+\frac{W_2(k-1)}{n}S_{y(2)}^2 \tag{2.10}$$

where $S_{y.x}=S_y^2+R^2S_x^2-2R\,S_{yx}$, $S_{y.x(2)}=S_{y(2)}^2+R^2S_{x(2)}^2-2R\,S_{yx(2)}$,

$$R=\frac{\bar{Y}}{\bar{X}}, \quad \beta=\frac{S_{yx}}{S_x^2}, \quad f=\frac{N-n}{N} \text{ and } f'=\frac{N-n'}{N}$$

In this case (S_y^2,S_x^2,S_{yx}) and $(S_{y(2)}^2,S_{x(2)}^2,S_{yx(2)})$ denote the population mean square of y,x and population covariance between y and x for the whole population and for the $N_2(=NW_2)$ non-response units of the population respectively. The correlation coefficient between y and x in the whole population is denoted by ρ.

BIAS AND MEAN SQUARE ERROR (MSE) OF THE PROPOSED CLASS OF ESTIMATORS

Under the regularity conditions imposed on $h(w,u_i)$, it may be seen that bias and mean square error of $T_i \ (i=1,2)$ will always exits. Now expanding $h(w,u_i)$ about the point $(\bar{Y},1)$ using Taylor's series up to second order partial derivatives, we have

$$\begin{aligned} T_i = h(\bar{Y},1) \ &+\ (w-\bar{Y})\ h_1(\bar{Y},1) \ +\ (u_i-1)\ h_{2(i)}(\bar{Y},1) \\ &+\ \frac{1}{2}\Big\{\left(w-\bar{Y}\right)^2\ h_{11}(w^*,u_i^*)\ +\ 2(w-\bar{Y})\ (u_i-1)\ h_{12(i)}(w^*,u_i^*) \\ &+\ (u_i-1)^2\ h_{22(i)}(w^*,u_i^*)\Big\}, \qquad \text{i=1, 2} \end{aligned} \tag{3.1}$$

Now using the condition

$$h(\bar{Y},1)\ =\ \bar{Y}\ h_1(\bar{Y},1)\text{, we have}$$

$$\begin{aligned} T_i = \bar{Y}\ h_1(\bar{Y},1) \ &+\ (w-\bar{Y})\ h_1(\bar{Y},1) \ +\ (u_i-1)\ h_{2(i)}(\bar{Y},1) \\ &+\ \frac{1}{2}\Big\{\left(w-\bar{Y}\right)^2\ h_{11}(w^*,u_i^*)\ +\ 2(w-\bar{Y})\ (u_i-1)\ h_{12(i)}(w^*,u_i^*) \\ &+\ (u_i-1)^2\ h_{22(i)}(w^*,u_i^*)\Big\} \qquad \text{i=1, 2} \end{aligned} \tag{3.2}$$

$$\begin{aligned} E(T_i-\bar{Y})\ =\ \bar{Y}(h_1(\bar{Y},1)-1)\ &+\ E(w-\bar{Y})\ h_1(\bar{Y},1)\ +\ E(u_i-1)\ h_{2(i)}(\bar{Y},1) \\ &+\ \frac{1}{2}E\Big\{\left(w-\bar{Y}\right)^2\ h_{11}(w^*,u_i^*)\ +\ 2(w-\bar{Y})\ (u_i-1)\ h_{12(i)}(w^*,u_i^*) \\ &+\ (u_i-1)^2\ h_{22(i)}(w^*,u_i^*)\Big\} \end{aligned} \tag{3.3}$$

which gives

$$\begin{aligned} Bias(T_i)\ =\ \bar{Y}(h_1(\bar{Y},1)-1)\ &+\ \frac{1}{2}\Big\{E\left(u_i-1\right)^2 h_{22(i)}\left(w^*,u_i^*\right)\ +\ 2E(w-\bar{Y}) \\ &\left(u_i-1\right)h_{12(i)}\left(w^*,u_i^*\right)\Big\} \qquad \text{i=1, 2} \end{aligned} \tag{3.4}$$

where $\quad w^* = \bar{Y} + \theta\left(w-\bar{Y}\right), \quad u_i^* = 1+\theta_i(u_i-1), \quad 0<\theta,\theta_i<1,$ i=1, 2

The expression for mean square error of T_i is obtained by putting

$h_{12(i)} = \bar{Y}^{-1} h_{2(i)}(\bar{Y},1)$ and neglecting $h_{22(i)}\left(\bar{Y},1\right)$ (as it is very small), we have $MSE(T_i)$ upto terms of order n^{-1} which is given by

$$\begin{aligned} MSE(T_i) &= \bar{Y}^2 (h_1(\bar{Y},1)-1)^2 + h_1^2(\bar{Y},1)\, E(w-\bar{Y})^2 + h_{2(i)}^2(\bar{Y},1)\, E(u_i-1)^2 \\ &\quad + 2(2h_1(\bar{Y},1)-1)\, h_{2(i)}(\bar{Y},1)\, E(w-\bar{Y})(u_i-1) \\ &= \bar{Y}^2 (h_1(\bar{Y},1)-1)^2 + Ah_1^2(\bar{Y},1) + B_i h_{2(i)}^2(\bar{Y},1) \\ &\quad +2Ci\,(2h_1(\bar{Y},1)-1)\, h_{2(i)}(\bar{Y},1), \qquad \text{i=1, 2,} \end{aligned} \tag{3.5}$$

for any sampling design.

The $MSE(T_i)$ will attain minimum value if

$$h_1(\bar{Y},1)_{opt} = \frac{\bar{Y}^2 B_i - 2C_i^2}{(\bar{Y}^2 + A)B_i - 4C_i^2} \qquad \text{i=1, 2} \tag{3.6}$$

and

$$h_{2(i)}(\bar{Y},1)_{opt} = \frac{C_i(A-\bar{Y}^2)}{B_i(\bar{Y}^2 + A) - 4C_i^2} \qquad \text{i=1, 2} \tag{3.7}$$

where

$$A = E(w-\bar{Y})^2 = E(\bar{y}^* - \bar{Y})^2,\ B_i = E(u_i-1)^2;\ B_1 = E\left(\frac{\bar{x}^*}{\bar{x}'} - 1\right)^2,\ \ B_2 = E\left(\frac{\bar{x}}{\bar{x}'} - 1\right)^2$$

$$C_i = E(u_i-1)(w-\bar{Y}), i=1,2;\ C_1 = E\left(\frac{\bar{x}^*}{\bar{x}'} - 1\right)(\bar{y}^* - \bar{Y}),\quad C_2 = E\left(\frac{\bar{x}}{\bar{x}'} - 1\right)(\bar{y}^* - \bar{Y}).$$

Now putting the optimum values of $h_1(\bar{Y},1)$ and $h_{2(i)}(\bar{Y},1)$ in (3.5), the expression for minimum $MSE(T_i)$ for any sampling design is given by

$$MSE(T_i) = \frac{4C_i^2(\bar{Y}^2 + A) + \bar{Y}^4 B_i(AB_i - C_i^2) + AB_i\left[\bar{Y}^2(AB_i - 6C_i^2) - AC_i^2\right]}{\left[\left(\bar{Y}^2 + A\right)B_i - 4C_i^2\right]^2} \tag{3.8}$$

Let us denote,

$$\bar{Y}_2 = \frac{1}{N_2}\sum_{i=1}^{N_2} Y_{2i},\quad \bar{X}_2 = \frac{1}{N_2}\sum_{i=1}^{N_2} X_{2i},\quad S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2,$$

$$S_{y(2)}^2 = \frac{1}{N_2 - 1}\sum_{i=1}^{N_2}(Y_{2i} - \bar{Y}_2)^2, S_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})(Y_i - \bar{Y})$$

and $S_{xy(2)} = \frac{1}{N_2 - 1}\sum_{i=1}^{N_2}(X_{2i} - \bar{X}_2)(Y_{2i} - \bar{Y}_2)$

Let ρ and $\rho_{(2)}$ denote the correlation coefficient between y and x for whole population and for the non-response group of the population. In case of SRSWOR, using the large sample approximation, we assume

$\bar{y}^* = \bar{Y}(1+\varepsilon_0)$, $\bar{x}^* = \bar{X}(1+\varepsilon_1)$, $\bar{x} = \bar{X}(1+\varepsilon_2)$ and $\bar{x}' = \bar{X}(1+\varepsilon_3)$

such that $E(\varepsilon_i) = 0$ and $|\varepsilon_i| < 1$, $\forall\ i = 0,1,2,3$ (3.9)

Now using SRSWOR, we have

$$E(\varepsilon_0^2) = \frac{V(\bar{y}^*)}{\bar{Y}^2} = \frac{1}{\bar{Y}^2}\left[\frac{f}{n}S_y^2 + \frac{W_2(k-1)}{n}S_{y2}^2\right],$$

$$E(\varepsilon_1^2) = \frac{V(\bar{x}^*)}{\bar{X}^2} = \frac{1}{\bar{X}^2}\left[\frac{f}{n}S_x^2 + \frac{W_2(k-1)}{n}S_{x2}^2\right],$$

$$E(\varepsilon_2^2) = \frac{V(\bar{x})}{\bar{X}^2} = \frac{f}{n}\frac{S_x^2}{\bar{X}^2}, \qquad E(\varepsilon_3^2) = \frac{V(\bar{x}')}{\bar{X}^2} = \frac{f'}{n'}\frac{S_x^2}{\bar{X}^2},$$

$$E(\varepsilon_0\varepsilon_1) = \frac{Cov(\bar{y}^*, \bar{x}^*)}{\bar{Y}\bar{X}} = \frac{1}{\bar{Y}\bar{X}}\left[\frac{f}{n}S_{yx} + \frac{W_2(k-1)}{n}S_{yx(2)}\right],$$

$$E(\varepsilon_0\varepsilon_2) = \frac{Cov(\bar{y}^*, \bar{x})}{\bar{Y}\,\bar{X}} = \frac{f}{n}\frac{S_{yx}}{\bar{Y}\,\bar{X}},$$

$$E(\varepsilon_0\varepsilon_3) = \frac{Cov(\bar{y}^*, \bar{x}')}{\bar{Y}\,\bar{X}} = \frac{f'}{n'}\frac{S_{yx}}{\bar{Y}\,\bar{X}}$$

$$E(\varepsilon_1\varepsilon_2) = \frac{Cov(\bar{x}^*, \bar{x})}{\bar{X}^2} = \frac{f}{n}\frac{S_x^2}{\bar{X}^2} \text{ and } E(\varepsilon_2\varepsilon_3) = \frac{Cov(\bar{x}, \bar{x}')}{\bar{X}^2} = \frac{f'}{n'}\frac{S_x^2}{\bar{X}^2} \quad (3.10)$$

Now using (3.10); we get the values of A', B_i' and $C_i'(i=1,2)$ in case of SRSWOR for $A, B_i(i=1,2)$ and $C_i(i=1,2)$ which are given as follows:

$$A' = E(w - \bar{Y})^2 = E(\bar{y}^* - \bar{Y})^2 = \left(\frac{f}{n}S_y^2 + \frac{W_2(k-1)}{n}S_{y2}^2\right), \quad (3.11)$$

$$B_1' = E(u_1 - 1)^2 = E\left(\frac{\bar{x}^*}{\bar{x}'} - 1\right)^2 = \frac{1}{\bar{X}^2}\left[\left(\frac{1}{n} - \frac{1}{n'}\right)S_x^2 + \frac{W_2(k-1)}{n}S_{x2}^2\right], \quad (3.12)$$

$$B_2^{'} = E(u_2 - 1)^2 = E\left(\frac{\bar{x}}{\bar{x}^{'}} - 1\right)^2 = \frac{1}{\bar{X}^2}\left[\left(\frac{1}{n} - \frac{1}{n^{'}}\right)S_x^2\right], \tag{3.13}$$

$$C_1^{'} = E(w - \bar{Y})(u_1 - 1)^2 = E(\bar{y}^* - \bar{Y})\left(\frac{\bar{x}^*}{\bar{x}^{'}} - 1\right)$$

$$= \frac{1}{\bar{X}}\left[\left(\frac{1}{n} - \frac{1}{n^{'}}\right)S_{yx} + \frac{W_2(k-1)}{n}S_{yx(2)}\right], \tag{3.14}$$

$$C_2^{'} = E(w - \bar{Y})(u_2 - 1)^2 = E(\bar{y}^* - \bar{Y})\left(\frac{\bar{x}}{\bar{x}^{'}} - 1\right) = \frac{1}{\bar{X}}\left[\left(\frac{1}{n} - \frac{1}{n^{'}}\right)S_{yx}\right] \tag{3.15}$$

Again using (2.11) to (2.15), we have

$$Bias_1(T_1) = \bar{Y}(h_1(\bar{Y},1) - 1) + \frac{1}{2}\left[A_1^{'}h_{22(1)}(w^*, u^*) + 2C_1^{'}h_{21(1)}\right], \tag{3.16}$$

$$Bias_1(T_2) = \bar{Y}(h_1(\bar{Y},1) - 1) + \frac{1}{2}\left[B_2^{'}h_{22(2)}(w^*, u^*) + 2C_2^{'}h_{21(2)}\right], \tag{3.17}$$

$$MSE(T_1) = \bar{Y}^2\left[\left(h_1(\bar{Y},1)\right) - 1\right]^2 + A^{'}h_1^2(\bar{Y},1) + B_1^{'}\, h_{2(1)}^2(\bar{Y},1) + 2C_1^{'}(2h_1(\bar{Y},1) - 1)\, h_{2(1)}(\bar{Y},1) \tag{3.18}$$

$$MSE(T_2) = \bar{Y}^2\left[\left(h_1(\bar{Y},1)\right) - 1\right]^2 + A^{'}h_1^2(\bar{Y},1) + B_2^{'}\, h_{2(2)}^2(\bar{Y},1) + 2C_2^{'}(2h_1(\bar{Y},1) - 1)\, h_{2(2)}(\bar{Y},1) \tag{3.19}$$

The $MSE(T_1)$ will be minimum for the optimum value of

$$h_1(\bar{Y},1) = \frac{\bar{Y}^2 B_1^{'} - 2C_1^{'2}}{(\bar{Y}^2 + A^{'})B_1^{'} - 4C_1^{'2}} \tag{3.20}$$

And
$$h_{2(1)}(\bar{Y},1) = \frac{C_1^{'}(A^{'} - \bar{Y}^2)}{B_1^{'}(\bar{Y}^2 + A^{'}) - 4C_1^{'2}} \tag{3.21}$$

Similarly, $MSE(T_2)$ will be minimum for the optimum values of $h_1(\bar{Y},1)$ and $h_{2(2)}(\bar{Y},1)$ which are given as follows:

$$h_1(\bar{Y},1) = \frac{\bar{Y}^2 B_2^{'} - 2C_2^{'2}}{(\bar{Y}^2 + A^{'})B_2^{'} - 4C_2^{'2}} \tag{3.22}$$

And
$$h_{2(2)}(\bar{Y},1) = \frac{C_2^{'}(A^{'} - \bar{Y}^2)}{B_2^{'}(\bar{Y}^2 + A^{'}) - 4C_2^{'2}} \tag{3.23}$$

Hence in case of SRSWOR, the minimum value of $MSE(T_i)$ for the optimum values of $h_1(\bar{Y},1)$ and $h_{2(i)}(\bar{Y},1)$ is given by

$$MSE(T_i) = \frac{4C_i^{'4}(\bar{Y}^2 + A^{'}) + \bar{Y}^4 B_i^{'}(A^{'}B_i^{'} - C_i^{'2}) + A^{'}B_i^{'}[(A^{'}B_i^{'} - 6C_i^{'2}) - A^{'}C_i^{'2}]}{[(\bar{Y}^2 + \mathrm{A}^{'})B_i^{'} - 4C_i^{'2}]^2} \quad (3.24)$$

CHOICE OF UNKNOWN CONSTANTS USED IN THE PROPOSED CLASS OF ESTIMATORS

The optimum values of $h_1(\bar{Y},1)$ and $h_{2(i)}(\bar{Y},1)$ give the solution for values of the unknown constants used in the estimators. However, sometimes these constants are in the form of some unknown parameters. In these situation, one may use the past data of the parameter for obtaining the optimum values of the constants, (Reddy (1978) or by estimating the parameters on the basis of sample values. It has been shown that upto the terms of order n^{-1}, the minimum value of the mean square error of the estimator is unchanged if we estimate the optimum values of the constants by using the sample values (Srivastava and Jhajj (1983)). Any parametric function $h(w,u_i)$ satisfying the condition (2.4) can generate a class of asymptotic. The class of such ratio type estimators is very large. Some member of the proposed class of estimators are given by

$$T_{i(1)} = W_0\ w u_i^{\alpha}, \ T_{i(2)} = \left[a_1 + a_2(u_i - 1)\right]w, \ T_{i(3)} = [a_1 w + a_2(u^{\alpha} - 1)]. \quad (4.1)$$

AN EMPIRICAL STUDY

One hundred and nine village /town /ward size population of urban area under police-station-Baria-Champua, Orissa has been taken under consideration from District Census Handbook, 1981, Orissa published by Govt. of India. The last 25% villages have considered as non-responding group of the population. Total population of the village is taken as study character y and the area of the village is taken as auxiliary character. The values of the parameters required in the study are as follows:

$\bar{Y} = 485.9174$, $\bar{X} = 256.3331$ $S_y = 320.2093$, $S_x = 156.4916$,, $S_{y(2)} = 236.9608$ $S_{x(2)} = 126.7178$, $\rho = 0.856$, $\rho_{(2)} = 0.768$

The problem under consideration is to estimate average total population of a village $\bar{Y}$, using auxiliary character as area of the village.In this problem we consider $T_{1(1)} = W_0\, w u_1^{\alpha}$ and $T_{2(1)} = a_1 w + a_2(u_1 - 1)$ as a member of class of estimators T_1 and T_2.The values of the constants used in these estimators are computed for different values of k and are given in table-1.

Table-1: The values of the constants for different values of k

Estimators	Constants	$1/k$		
		1/2	1/3	1/4
$T_{1(1)}$	w_0	1.003	1.004	1.0008
	α	-0.8885	-0.8521	-0.8483
$T_{2(1)}$	a_1	0.99	0.99	0.99
	a_2	-441.047	-441.047	-441.047

Table-2: Relative Efficiency of the estimators (in %) w. r. to $\bar{y}^*$ for different values of k $(n' = 70,\ n = 40)$

Estimators	$1/k$		
	1/4	1/3	1/2
$\bar{y}^*$	100.00 (2663.17)	100.00 (2315.46)	100.00 (1967.75)
t_1	211.72 (1257.89)	208.01 (1113.17)	205.63 (956.93)
t_2	143.24 (1859.28)	153.18 (1511.57)	169.07 (1163.85)
t_3	203.36 (1309.60)	202.05 (1145.98)	200.31 (982.86)
t_4	143.04 (1861.87)	152.92 (1514.16)	168.70 (1166.45)
$T_{1(1)}$	215.52(1235.68)	210.04 (1102.36)	209.77 (938.01)
$T_{2(1)}$	144.88 (1838.18)	154.63 (1497.40)	170.13 (1156.60)

From table-2, we see that for fixed values of n' and n, the member $T_{1(1)}$ and $T_{2(1)}$ of the proposed class of estimators T_1 and T_2 are more efficient than the corresponding estimators (t_1, t_3) and (t_2, t_4) respectively. The values of MSE of $T_{1(1)}, T_{2(1)}$, t_1, t_2, t_3 and t_4 decreases as the value of k decreases and is always less than the variance of $\bar{y}^*$. The relative efficiency of (t_1, t_3) is less than the relative efficiency of $T_{1(1)}$ and the relative efficiency of (t_2, t_4) is less than the relative efficiency of $T_{2(1)}$ for different values of k. Hence we prefer to use the class of estimator T_1 in comparison to t_1 and t_3 and T_2 in comparison to t_2 and t_4 according to the situation.

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