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SHRINKAGE ESTIMATOR AND TESTIMATORS FOR SHAPE PARAMETER OF CLASSICAL PARETO DISTRIBUTION

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ABSTRACT

This paper considers the problem of estimation of shape parameter of classical Pareto distribution under apriori information in the form of a point guess value when the scale parameter is unknown. It is seen that the proposed estimator behaves better than the usual unbiased estimator when the prior point guess is close to the true value. This conclusion encourages deciding first whether point guess is closed to the true value or not and accordingly one should think of using the proposed estimator or the usual unbiased estimator. Hence, some testimators are defined through the use of preliminary test procedure. The properties of proposed estimators have been studied in terms of bias and mean square error.

Keywords and phrases: Pareto distribution, Shrinkage Estimator, Preliminary Test Estimators, Relative Bias, Relative Efficiency

INTRODUCTION

Classical Pareto distribution was initially introduced as a model for distribution of income exceeding a certain limit. But George Zipf (1949) commented that many variables associated with economic and social phenomena follow a Pareto distribution. It has also been used in connection with reliability theory and survival analysis (see Davis and Feldstein, 1979; Abdel-Ghaly, Attia and Aly, 1998 etc.). This distribution has played an important role in a variety of problems such as economic studies of income (Champerowne, 1953; Mandelbrot, 1960 etc.), size of cities and firms (Steindl, 1965), business mortality (Lomax, 1954), service time in queuing systems (Harris, 1967).

The probability density function of the classical Pareto distribution is

$$f(X;\sigma,a) = a\,\sigma^{a}\,X^{-(a+1)}\,; \qquad X \ge \sigma, \quad a > 0$$

where σ and a are the scale and shape parameter respectively.

Muniruzzaman (1957) has discussed maximum likelihood estimation of several measures of location for classical Pareto distribution. Quandt (1966) has obtained different estimators for the parameters of this distribution using the method of maximum likelihood, method of least square and quantile method and discussed their properties. The maximum likelihood estimators for the parameters of the distribution obtained by Quandt (1966) are

$$\hat{\sigma} = X_{(1)} = \min(X_1, X_2, X_3, \cdots, X_n) \quad \text{and}$$

$$\hat{a} = \left[\frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{X_i}{X_{(1)}}\right)\right]^{-1}$$

It may be noted here that $\hat{\sigma}$ and $\hat{a}$ are jointly sufficient and are consistent estimates for σ and a respectively (in fact strongly consistent).

Malik (1970b) derived the distribution of maximum likelihood estimates of scale and shape parameters and showed that they are independently distributed. The result is implicit in Muniruzzaman (1957) (see also Baxter (1980). Saksena and Johnson (1984) have obtained the unique minimum variance unbiased estimator of the scale and shape parameter based on complete sufficient statistics (see also Baxter (1980) and Likes (1969)).

It can be shown that $\hat{\sigma} = X_{(1)}$ follows Pareto distribution with parameters σ and na whereas $2na/\hat{a}$ follows a chi-square distribution with (2n−2) degree of freedom. It may further be noted that these maximum likelihood estimators are biased. However, the biases can be easily estimated and hence, the unbiased estimator of σ and a and their corresponding mean square error can easily be obtained as

$$\hat{\sigma}_u = [1-(n-1)^{-1}\hat{a}^{-1}]\hat{\sigma}$$

$$MSE(\hat{\sigma}_u) = \sigma^2 a^{-1}(n-1)^{-1}(an-2)^{-1} \quad \text{and}$$

$$\hat{a}_u = \frac{n-2}{n}\hat{a}$$

$$MSE(\hat{a}_u) = a^2(n-3)^{-1}$$

Shrinkage technique proposed by Thompson (1968a) is one of the most popular technique for improving the existing estimator T of parameter θ when a prior guess θ_0 is available to us with confidence k by defining the estimator

$$T_{Th} = k\,\theta_0 + (1-k)T \,.$$

It has been noted by various authors that T_{Th} performs better than T if θ is close to θ_0 and k is taken large. However for larger deviation of θ_0 from θ, T_{Th} may be worse. The range in which T_{Th} performs better than T can be increased by taking k small. Thus, mostly it is often concluded that if it is expected that θ is in vicinity of θ_0, one should use T_{Th} with large k whereas if θ is close but not in vicinity of θ_0, one can use the T_{Th} with small k. In order to check whether θ is in vicinity of θ_0 or not, one can use a preliminary test for testing $H_0 : \theta = \theta_0$ and if hypothesis is not rejected, we use T_{Th} otherwise T may be used. It may be noted that preliminary test shrinkage estimator provides protection against the use of less efficient estimators.

It is clear from the above discussion that if the choice of confidence k in the guess value is in accordance with the real situation, the shrinkage estimator performs better than the usual estimator. Thus, instead of taking k to be a fixed constant in the shrinkage estimator one should take it as a weight (lying between 0 and 1) which takes large value if θ is expected to be close to θ_0 and small value if θ is away from θ_0. In other words k can be taken as a continuous function of some suitable statistics so that its value monotonically decreases as $(\theta-\theta_0)$ is expected to increase. Attempts have been made by various authors for the choice of such continuous weighting function. Mehta and Srinivasan (1971) proposition of choice of the weight function as function of preliminary test statistics has been considered by various authors (see Pandey and Mishra, 1991, 92).

This paper aims to consider the Mehta and Srinivasan's proposition for the estimation of shape parameter of Pareto distribution when scale parameter is also unknown. The estimator, thus obtained, will be studied for its performance as compared to the usual unbiased estimator. Lastly recommendation for the use of estimators will be made.

ESTIMATION OF THE SHAPE PARAMETER WHEN SCALE PARAMETER IS UNKNOWN

Let us consider that a random sample of size n is drawn from a Pareto distribution. For testing a hypothesis $H_0 : a = a_0$, the test procedure proposed by Muniruzzaman (1957) is based on the statistic $w = 2(n-2)a_0/\hat{a}_u$ where $\hat{a}_u$ is the usual unbiased estimate of a. It may noted here that test statistics follows chi-square distribution with 2(n–1) degrees of freedom under H_0. Let us consider $\phi(w) = de^{-bw}$ where d and b are positive constants such that $0 \le d \le 1$ and $b > 0$. For every such choice of d and b, it may be noted that $\phi(w)$ is continuous function of w and

$\phi(w) \to 0$ if $w \to \infty$

$\phi(w) \to d$ if $w \to 0$

In other words as w increases $\phi(w)$ decreases. A large of w indicates that $H_0 : a = a_0$ may be wrong. Therefore, we may propose a modified shrinkage estimator as

$$T = \phi(w)a_0 + (1 - \phi(w))\,\hat{a}_u \quad (2.1)$$

A somewhat similar estimator for exponential distribution has been considered by Pandey and Mishra (1991,1992).

Bias and Mean Square Error of Estimator T

Bias of the proposed estimator is defined as

$$\text{Bias}(T) = E(T) - a$$

It is easy to verify that the bias of T is obtained as follows:

$$\text{Bias}(T) = ad[\delta\psi^{n-1} - \psi^{n-2}] \quad (2.3)$$

and

$$\text{Relative bias }(T) = \frac{\text{Bias }(T)}{a}$$

$$= d[\delta\psi^{n-1} - \psi^{n-2}] \quad (2.4)$$

where $\delta = \dfrac{a_0}{a}$ and $\psi = (2b\delta + 1)^{-1}$.

The MSE of T can be expressed as

$$\text{MSE}(T) = E(T^2) - 2a\,E(T) + a^2 \quad (2.5)$$

Further, expression of mean square error of the proposed estimator can easily be obtained as

$$\frac{\text{MSE}(T)}{a^2} = (n-3)^{-1} + d^2(n-2)(n-3)^{-1}\,\psi^{*n-3}$$

$$+\ d^2\delta^2\psi^{*n-1} - 2d^2\delta\,\psi^{*n-2} - 2d(n-2)(n-3)^{-1}\,\psi^{n-3}$$

$$+\ 2d\,\delta\,\psi^{n-2} + 2d\psi^{n-2} - 2d\,\delta\,\psi^{n-1} \tag{2.6}$$

where $\psi^* = (4b\delta+1)^{-1}$.

It may be seen as $$\frac{MSE(\hat{a}_u)}{a^2} = (n-3)^{-1}.$$

Therefore relative efficiency of T with respect to usual unbiased estimator $\hat{a}_u$ is defined as

$$R.E.(T,\hat{a}_u) = \frac{MSE(\hat{a}_u)}{MSE(T)}$$

$$= [1 + d^2(n-2)\psi^{*n-3} + d^2\delta^2(n-3)\,\psi^{*n-1} - 2d^2(n-3)\,\delta\psi^{*n-2}$$

$$-\ 2d(n-2)\psi^{n-3} + 2d(n-3)\,\delta\psi^{n-2}$$

$$+\ 2d(n-3)\,\psi^{n-2} - 2d(n-3)\,\delta\psi^{n-1}]^{-1} \tag{2.7}$$

PRELIMINARY TEST ESTIMATORS FOR SHAPE PARAMETER

The use of preliminary test for subsequent estimation of parameter has been proposed for the first time by Bancroft (1944) and the estimators thus obtained are popularly known as preliminary test estimators. A number of authors have used preliminary test estimators in various situations. For detail bibliography readers are referred to Bancroft and Han (1977).

PRELIMINARY TEST SHRUNKEN ESTIMATOR $\hat{a}_{PTS1}$

It may be noted from the previous section that if a is close to point guess a_0 the proposed estimator behaves better than usual unbiased estimator $\hat{a}_u$ but it may be worse otherwise. Let us consider the situation that the point guess a_0 is either equal to the true value a or less than that. Hence, we propose to make a choice between T and $\hat{a}_u$ for their use based on preliminary test. Needless to mention that large value of test statistic $w = 2(n-2)\frac{a_0}{\hat{a}_u}$ indicate that $H_0 : a = a_0$ may be rejected in favors of $H_1 : a > a_0$. Therefore, we propose the following preliminary test shrunken estimator for its use in such situations:

$$\hat{a}_{PTS1} = \begin{cases} T & w \le c \\ \hat{a}_u & w \ge c \end{cases} \tag{3.1}$$

where choice of c depends on the level of significance (α) of preliminary test.

Therefore, the expression for the bias and relative bias of $\hat{a}_{PTS1}$ can be given as

$$\text{Bias}(\hat{a}_{PTS1}) = a[d\delta\psi^{n-1} G_X(n-1) - d\psi^{n-2} G_X(n-2)] \tag{3.2}$$

and

$$\text{R.B}(\hat{a}_{PTS1}) = d[\delta\psi^{n-1} G_X(n-1) - \psi^{n-2} G_X(n-2)] \tag{3.3}$$

where ψ and δ has already been defined in (2.4), $G_X(\lambda)$ is incomplete gamma function defined as.

$$G_X(\lambda) = \frac{1}{\Gamma\lambda} \int_0^X t^{\lambda-1} e^{-t}\, dt$$

and $$X = c(b + \frac{1}{2\delta}).$$

Now,

$$\text{MSE}(\hat{a}_{PTS1}) = E(\hat{a}^2_{PTS1}) - 2aE(\hat{a}_{PTS1}) + a^2 \tag{3.4}$$

which can easily be evaluated as

$$\begin{aligned}\frac{\text{MSE}(\hat{a}_{PTS1})}{a^2} = \Bigg[& \frac{1}{(n-3)} + \frac{(n-2)}{(n-3)} d\{d\psi^{*n-3} G_{X_1}(n-3) - 2\psi^{n-3} G_X(n-3)\} \\ & + d\delta\{(d\delta\psi^{*n-1} G_{X_1}(n-1) - 2d\psi^{*n-2} G_{X_1}(n-2)) \\ & + 2(\psi^{n-2} G_X(n-2) + \psi^{n-1} G_X(n-1))\} \\ & + 2d\psi^{n-2} G_X(n-2)\Bigg]\end{aligned} \tag{3.5}$$

Where ψ^* is defined in (2.6) and $X_1 = c(2b + \frac{1}{2\delta})$.

Therefore the expression for the relative efficiency of $\hat{a}_{PTS1}$ with respect to usual unbiased estimator $\hat{a}_u$ defined as

$$\text{R.E.}(\hat{a}_{PTS1}, \hat{a}_u) = \frac{\text{MSE}(\hat{a}_u)}{\text{MSE}(\hat{a}_{PTS1})},$$

can easily be obtained as follows :

$$\begin{aligned}\text{R.E.}(\hat{a}_{PTS1}, \hat{a}_u) = [1 + (n-2)\, d\{d\psi^{*n-3}\, G_{X_1}(n-3) - 2\psi^{n-3}\, G_X(n-3)\} \\ + (n-3)\, d\delta\{d\delta\{(\psi^{*n-1}\, G_{X_1}(n-1) - 2d\psi^{*n-2}\, G_{X_1}(n-2)) \\ + 2(\psi^{n-2}\, G_X(n-2) - \psi^{n-1}\, G_X(n-1))\} \\ + 2(n-3)d\psi^{n-2}\, G_X(n-2)]^{-1}\end{aligned} \tag{3.6}$$

PRELIMINARY TEST SHRUNKEN ESTIMATOR $\hat{a}_{PTS2}$

In the previous section we proposed a preliminary test shrunken estimator if guess value is either equal to or less than the true value. But in practice, deviation of the guess value cannot be ruled out in either direction. Therefore in such situations, $H_0 : a = a_0$ may be rejected at α percent level of significance if $w \le c_1$ or $w \ge c_2$, where c_1 and c_2 are such that

$$P_{H_0}[w \le c_1 \cup w \ge c_2] = \alpha,$$

c_1 and c_2 may be values of unbiased partition or partition with equal tail area. Hence, we may propose the preliminary test shrunken estimator for the shape parameter under this situation as

$$\hat{a}_{PTS2} = \begin{cases} T & c_1 \le 2(n-2)\frac{a_0}{\hat{a}_u} \le c_2 \\ \hat{a}_u & \text{otherwise} \end{cases} \tag{3.7}$$

BIAS and MSE of $\hat{a}_{PTS2}$

The bias of $\hat{a}_{PTS2}$ is defined as

$$Bias(\hat{a}_{PTS2}) = E(\hat{a}_{PTS2}) - a$$

Therefore the bias of $\hat{a}_{PTS2}$ is as follows:

$$\text{Bias}(\hat{a}_{PTS2}) = da[\delta\psi^{n-1}\{G_{X_2}(n-1) - G_{X_3}(n-1)\}$$

$$-\psi^{n-2}\{G_{X_2}(n-2)\ -\ G_{X_3}(n-2)\}] \qquad (3.8)$$

and the expression for the relative bias is

$$R.\,Bias(\hat{a}_{PTS2})\ =\ \frac{Bias(\hat{a}_{PTS2})}{a}$$

$$=\ d[\delta\psi^{n-1}\{G_{X_2}(n-1)\ -\ G_{X_3}(n-1)\}$$

$$-\ \psi^{n-2}\{G_{X_2}(n-2)\ -\ G_{X_3}(n-2)\}] \qquad (3.9)$$

where, $X_2\ =\ c_2(b+\frac{1}{2\delta})$ and $X_3\ =\ c_1(b+\frac{1}{2\delta})$.

Now

$$MSE(\hat{a}_{PTS2})\ =\ E(\hat{a}^2_{PTS2})\ -\ 2aE(\hat{a}_{PTS2})\ +\ a^2$$

Evaluation of various terms in the above expression is quite simple and after simplification it finally reduces to

$$\frac{MSE(\hat{a}_{PTS2})}{a^2}\ =\ \left\{\frac{(n-2)}{(n-3)}\ +\ \frac{(n-2)}{(n-3)}d^2\psi^{*n-3}[G_{X_2'}(n-3)-G_{X_1'}(n-3)]\right.$$

$$+\ d^2\delta^2\psi^{*n-1}[G_{X_2'}(n-1)\ -\ G_{X_1'}(n-1)]$$

$$-\ 2d^2\,\delta\,\psi^{*n-2}[G_{X_2'}(n-2)\ -\ G_{X_1'}(n-2)]$$

$$-\ \frac{2(n-2)}{(n-3)}d\,\psi^{n-3}[G_{X_2}(n-3)-G_{X_3}(n-3)]$$

$$+\ 2d\psi^{n-2}[G_{X_2}(n-2)\ -\ G_{X_3}(n-2)](\delta+1)$$

$$\left.-\ 2d\,\delta\,\psi^{n-1}[G_{X_2}(n-1)\ -\ G_{X_3}(n-1)]-1\right\} \qquad (3.10)$$

where

$$X_1'\ =\ c_1\left(2b+\frac{1}{2\delta}\right) \qquad \text{and} \qquad X_2'\ =\ c_2\left(2b+\frac{1}{2\delta}\right).$$

Hence, the relative efficiency of $\hat{a}_{PTS2}$ with respect to usual unbiased estimation $\hat{a}_u$ is

$$R.E.(\hat{a}_{PTS2}, \hat{a}_u) = \frac{MSE(\hat{a}_u)}{MSE(\hat{a}_{PTS2})}$$

$$= \{1 + (n-2)\, d^2 \psi^{*n-3} [G_{X_2'}(n-3) - G_{X_1'}(n-3)]$$

$$+ (n-3)\, d^2 \delta^2 \psi^{*n-1} [G_{X_2'}(n-1) - G_{X_1'}(n-1)]$$

$$- 2(n-3)\, d^2 \delta\, \psi^{*n-2} [G_{X_2'}(n-2) - G_{X_1'}(n-2)]$$

$$- 2(n-2)\, d\psi^{n-3} [G_{X_2}(n-3) - G_{X_3}(n-3)]$$

$$+ 2(n-3)(\delta+1)\, d\psi^{n-2} [G_{X_2}(n-2) - G_{X_3}(n-2)]$$

$$- 2(n-3)\, d\, \delta\, \psi^{n-1} [G_{X_2}(n-1) - G_{X_3}(n-1)]\}^{-1} \quad (3.11)$$

DISCUSSION FOR ESTIMATOR T

Now we will study the performance of the proposed estimator in comparison to the usual unbiased estimator. It may be noted here that relative bias and relative efficiency of the proposed estimator as compared to usual unbiased estimator is function of n, δ, d and b. Out of these parameters n is the sample size and d and b are the constants involved in the weight used in defining the shrinkage estimator. The values of these are to be fixed by the experimenter. Whereas, δ is ratio of guessed and actual values of the parameter and thus it is beyond the control of the experimenter. Therefore, to study the behavior of the relative bias and relative efficiency, we calculated the values of these for various values of n, δ, d and b. The values considered by us are as follows:

n = 5(5)15

$\delta = \frac{a_0}{a}$ = .25(.25)3.75

d = 0.2, 0.3, 0.4, 0.6, 0.8

And b = .001, 0.01, 0.20, 0.40, 0.60, 0.80, 1.00.

The results thus obtained are summarized in the form of graphs. Due to paucity of space and similarity in the trend of relative bias and relative efficiency, all the figures are not reproduced here only partial results are shown in **figures 1-14.**

Figures 1-4 shows the variation in the values of δ when d and b are fixed and different curve show the relative biases for different values of n. It may be noted from these figures that relative bias is negative for $\delta < 1$. As δ increases, it may be noted that bias increases but the magnitude decreases till it becomes zero for some value of δ. Such a value of δ is closer to 1 when b is small and greater than 1 when b is large. It may also be noted that for small values of b, the relative bias seems to increase by linear rate with δ but for large or moderate values of b, positive bias increases with a much slower rate.

Further the effect of d may also be noticed from comparison of **figure 1 with 3** and **2 with 4**. It may be easily seen that as d increases, the relative bias increases for both small as well as moderate values of b but it may also be noted that the magnitude of the relative bias is less for the moderate value of b. It is further noted that as the sample size increases, the relative bias decreases but for small value of b, the magnitude of relative bias decreases with slower rate as compared to that for large values of b and δ.

Figures 5-7 shows the variation in the relative bias for the variation in the values of b when the sample size and the ratio of the guess and true value is fixed and different curve show the relative bias for different value of d. It may be easily seen from these figures that for large choice of b the magnitude of relative bias is negligibly small. The choice of d should be small in such cases. It may further be noted that if δ is small except for small values of b the magnitude of the relative bias is quite small. However, if δ is 1 or greater than one, it was noted that for small choices of d, the magnitude of the relative bias is quite small except for a range of values of b around 0.2.

Figures 8-11 summarizes the results for relative efficiency of the proposed estimator. It can be noted that as sample size increases, the relative efficiency in general decreases. Though, the trend of variation for change in value of δ remains more or less same. As δ increases, the relative efficiency in general decreases but for moderate or large choice of b it is often noted to be larger than one. However for small choices of b and d and for large values of δ, it is observed that the proposed estimator may have relative efficiency less than 1. In such a case the maximum relative efficiency is seen for $\delta = 1$. It is further noted that for moderate values of d and b, the relative efficiency is seen to be more than one for almost all the considered values of other parameter. As δ becomes large, the relative efficiency comes closer to one. It is worthwhile to mention that greater gains are seen for a sub parameter space around $\delta = 1$ with small choices of d and b. However, for moderate choices of d and b, the range in which the relative efficiency is greater than one increases but the magnitude of relative efficiency decreases.

Figures 12-14 show the variations in relative efficiency due to variation in the values of b. It may be noted from these figures that if δ is small (< 0.50), the greater gain in efficiency is obtained by taking d to be moderate values namely $0.3 \leq d \leq 1$ and b to be small i.e. around 0.2. However, if δ is large, moderate choices of d and b provides relative efficiency greater than one in most of the cases though the gain is smaller.

DISCUSSION FOR PRELIMINARY TEST ESTIMATORS

To study the performance of the proposed estimators, we have considered the arbitrary value of d and b namely d = 0.3 and b = 0.01 for the sample size n = 5, 10, 21 and δ = 0.25(0.25)2. The relative biases of the proposed estimators and relative efficiencies as compared to the usual unbiased estimator, for the above mentioned values of the parameters are calculated and the results are summarized in the form of figures.

Performance of the $\hat{a}_{PTS1}$: Figures 15-17 shows the variation in relative bias for the variation in the values of δ when n, d and b are fixed and different curve show the relative bias for the different level of significance namely α = 1%, 5% and 10%. It may be noticed that relative bias is negative for $\delta < 1$. As δ increases the relative bias increases but in magnitude it decreases. Ultimately for a value of δ close to 1 it becomes zero. For further increase in the value of δ, relative bias becomes positive and increases but after a moderate value of δ ($1.75 \leq \delta \leq 2.0$) a further increase in δ results a decrease in bias. It is worthwhile to remark here that the proposed estimator has been defined for the situation when $\delta \leq 1$. Therefore, we see that the proposed estimator has often negative relative bias and becomes negligibly small for δ close to one. It may also be noted from the comparison of figures that as sample size increases the magnitude of relative bias decreases. The effect of change of preliminary level of significance has little effect on the relative bias, although a nominal decrease in the relative bias can be noted for preliminary level of significance α = 1% as compared to other considered values of α.

Figures 18–20 show the variation in relative efficiency for the variation in the values of δ. It may be noted that proposed estimator performs better than the usual unbiased estimator for all considered values of δ. It may be easily seen that maximum gain is obtained when guess value is in the vicinity of true value. Further, it may be noted that for small level of significance the relative efficiency is more than those for large values of it except when δ is not much larger than 1 (i.e. often $\delta < 1.5$). The effect of increasing the sample size is also easy to see from the comparison of figures and it may be noted that as the sample size increases the gain as well as the effective interval (i.e. range of δ for which relative efficiency is greater than one) decreases.

Performance of the $\hat{a}_{PTS2}$: Figures 21-23 shows the variation in relative bias of $\hat{a}_{PTS2}$ for the variation in the values of δ and different curves show the relative bias for the different level of significance. It may be noted from the figures that for small values of δ (< 1), the relative bias is often negative, although it is negligibly small for values of δ close to zero. As δ increases the relative bias decreases (resulting into an increase in the magnitude of bias) initially reaches to a minimum (a maximum in magnitude) for some values of δ and then starts increasing (decreasing in magnitude). It becomes zero for some values δ close to $\delta = 1$. For further increase in δ, the relative bias increases becomes maximum for some value of δ and then decreases for further increase in δ. In this way we see that for δ close to 1, the magnitude of relative bias is negligibly small and δ move away from 1 magnitude of bias increases. However for larger deviations of δ, the relative bias is again negligibly small in magnitude. It may also be noted that the magnitude of bias is smaller for larger sample size and larger values of preliminary level of significance.

Figures 24-26 show the variation in relative efficiency for the variation in the values of δ. The proposed estimator performs better than usual unbiased estimator if δ is close to one. It may be noted that for sub-region of parameter space around $\delta = 1$, the relative efficiency is greater than one. Such a region may be termed as effective interval for the proposed estimator. It may also be noted that for some values of δ around $\delta = 1$, the relative efficiency curve has a maximum and as δ moves away from this point in either direction, the relative efficiency is noted to continue, even when δ moves out of effective interval. After attaining a minimum value a slight increase in relative efficiency is noted for further departure of δ on either side. In other words, we see from the figures that if $0.65 \leq \delta \leq 1.60$ for all the considered situations the relative efficiency is greater than one. The range of δ for which relative efficiency is greater than one is larger for small sample sizes and small level of significance as compared to large sample sizes and large values of level of significance. However the magnitude of gain is more for larger sample sizes and small level of significance.

CONCLUSION

From discussion of estimator T we may conclude that the proposed estimator performs better than usual unbiased estimator for proper choice of the constants d and b. If one is confident that the true value is expected to be close to the guess value, one may take d and b small namely $0.3 \le d \le 0.6$ and $0.001 \le b \le 0.01$. However, if it is suspected that the true value may be different from the guessed value, then a moderate value of b and small values of d provides an estimator which is more efficient than usual unbiased estimator in the sense that it has smaller mean square error and negligible bias. In brief, we may therefore recommend for the use of proposed estimator with small choices of d and moderate choice of b.

On the basis of discussion of preliminary test estimator we notice that one sided preliminary test estimator which is justified only when guess value is always less than the true value ($\delta < 1$), performs better than the usual unbiased estimator for $\delta < 1$. Moreover even if $a_0 > a$, the efficiency of proposed one sided preliminary test estimator performs better for $\delta \le 1.65$. Thus if we are confident that guess value is always less than the true value we may recommend the use of this estimator with the choices of d and b as mentioned in previous section. However, if the guess can either be less than or greater than the true value, two sided preliminary test estimator is logically more justified and can be recommended for its use. It may worthwhile to mention that the use of two sided preliminary test estimator either provides a better estimate than the usual unbiased estimator or nominal losses in a very small parameter space. In rest of parameter space it is as efficient as usual unbiased estimator. It is also concluded from the above discussion that 1% level of significance may be recommended for the use of preliminary test estimator because it provides greater gain and lesser losses. Needless to mention that proposed procedure of estimates is more beneficial for its use if sample size is small.

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Fig 1: Relative Bias of the Estimator T when d=0.2 and b=0.001

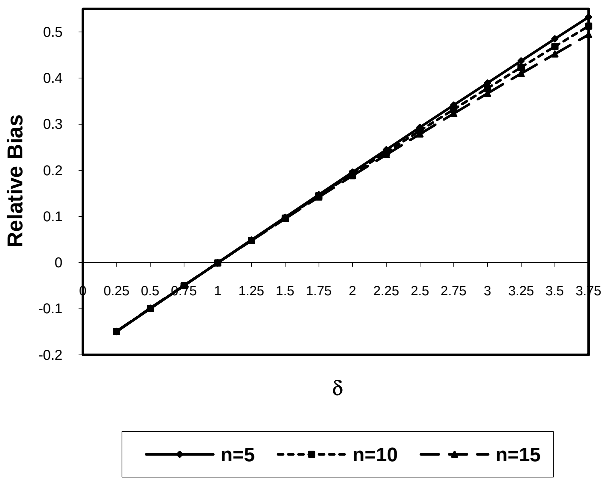

Fig 2: Relative Bias of the Estimator T when d=0.2 and b=0.2

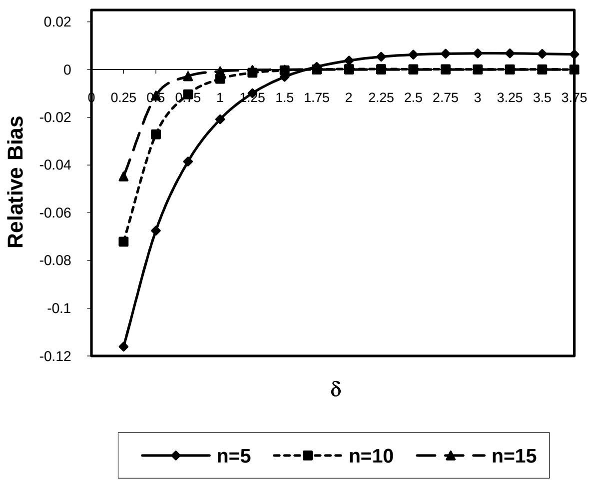

Fig 3: Relative Bias of the Estimator T when d=0.4 and b=0.001

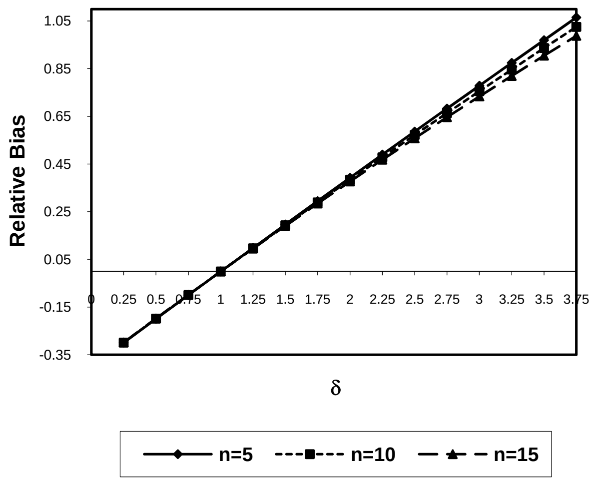

Fig 4: Relative Bias of the Estimator T when d=0.4 and b=0.2

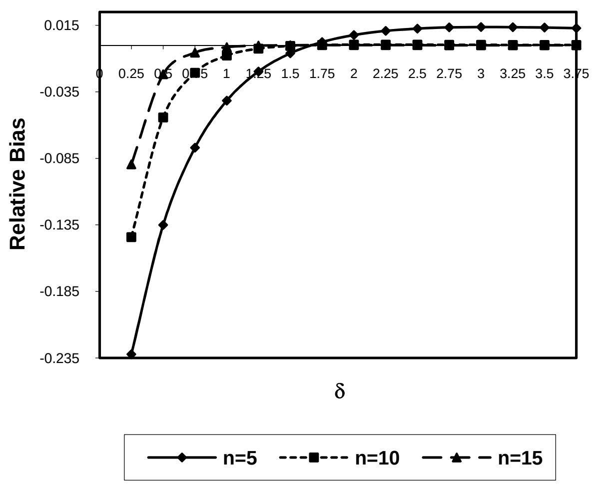

Fig 5: Relative Bias of the Estimator T for n=5 and d=0.5

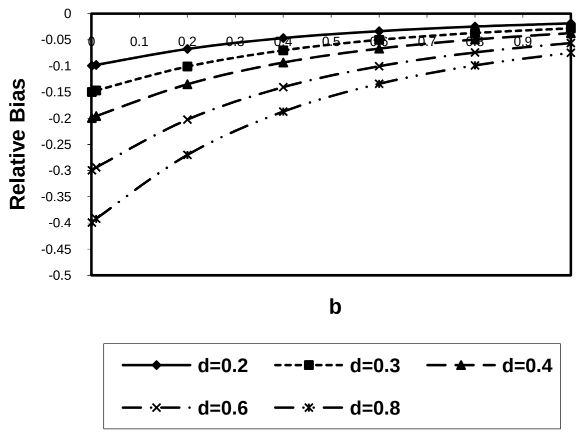

Fig 6: Relative Bias of the Estimator T for n=5 and d=0.75

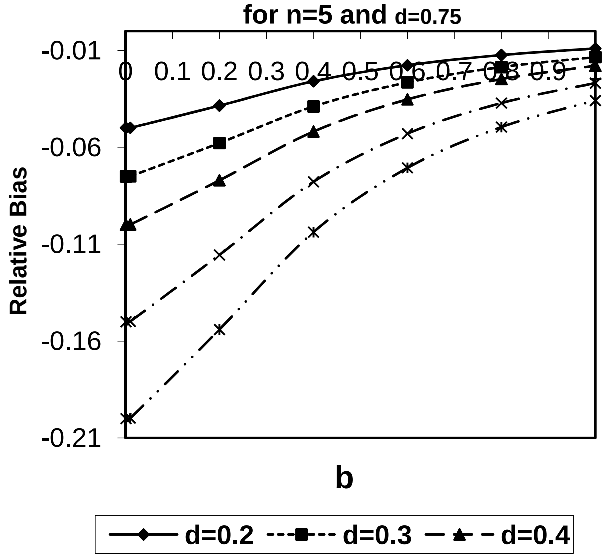

Fig 7: Relative Bias of the Estimator T for n=5 and d=1.0

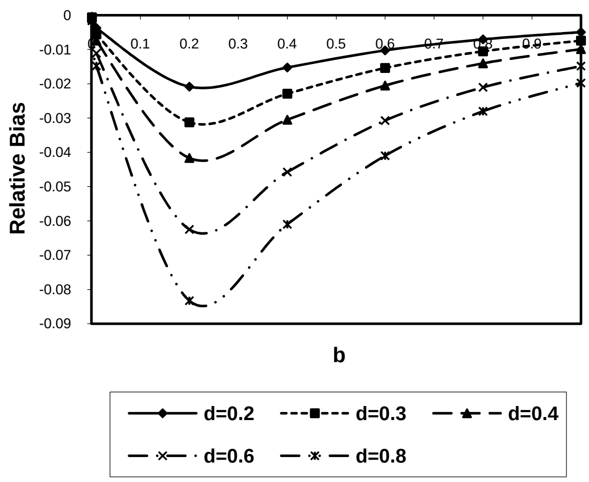

Fig 8: Relative Efficiency of estimator T with respect to âu when d=0.2 and b=0.001

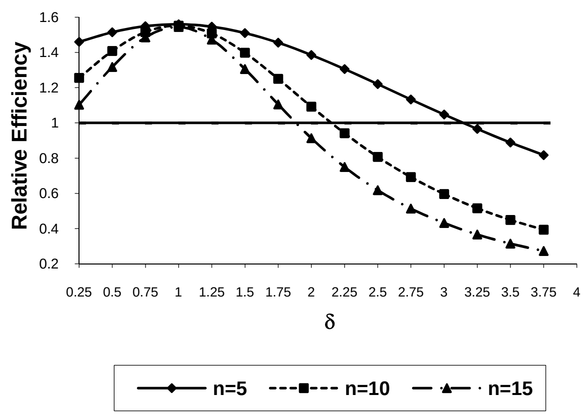

Fig 9: Relative Efficiency of estimator T with respect to âu when d=0.2 and b=0.2

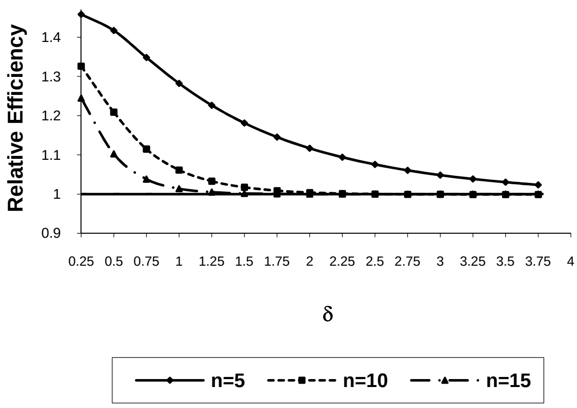

Fig 10: Relative Efficiency of estimator T with respect to âu when d=0.4 and b=0.001

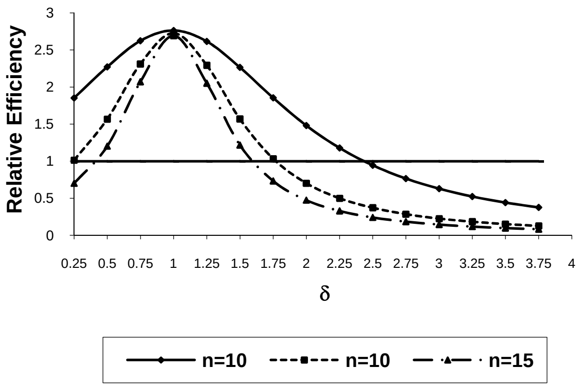

Fig 11: Relative Efficiency of estimator T with respect to âu when d=0.4 and b=0.2

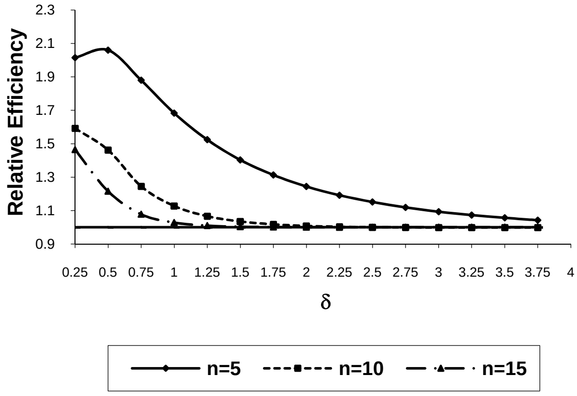

Fig 12: Relative Efficiency of estimator T with respect to âu when n=5 and d=.5

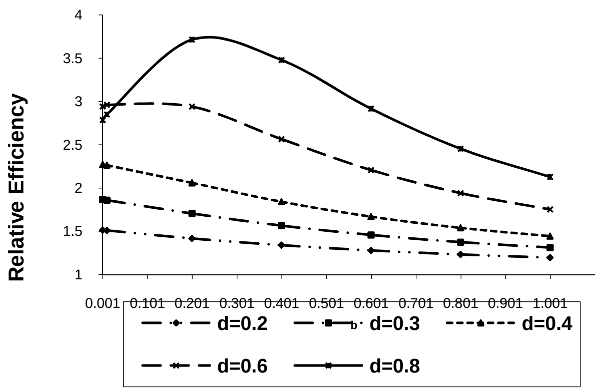

Fig 13: Relative Efficiency of estimator T with respect to âu when n=5 and d=0.75

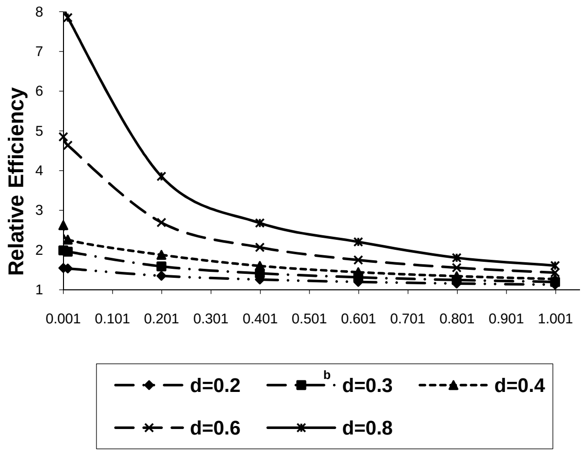

Fig 14: Relative Efficiency of estimator T with respect to âu when n=5 and d=1

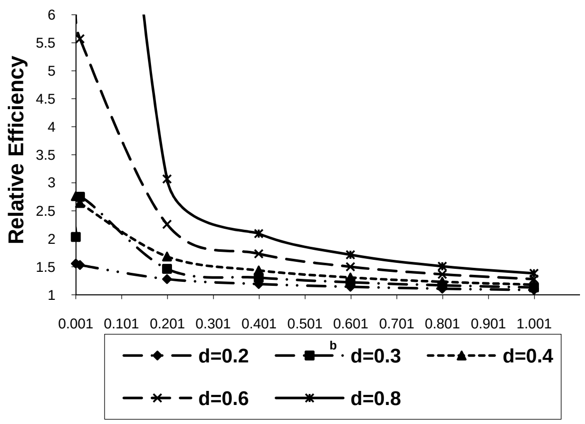

Fig 15: Relative Bias of Preliminary Test Estimator $\hat{a}_{PTS1}$ when n=5, d=0.3 and b=0.01

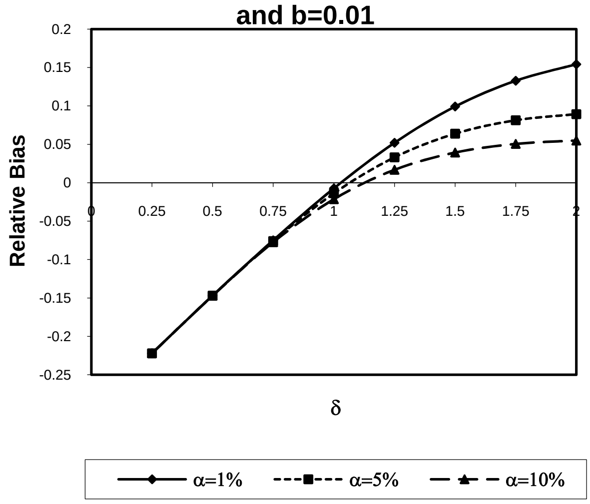

Fig 16: Relative Bias of Preliminary Test Estimator $\hat{a}_{PTS1}$ when n=10, d=0.3 and b=0.01

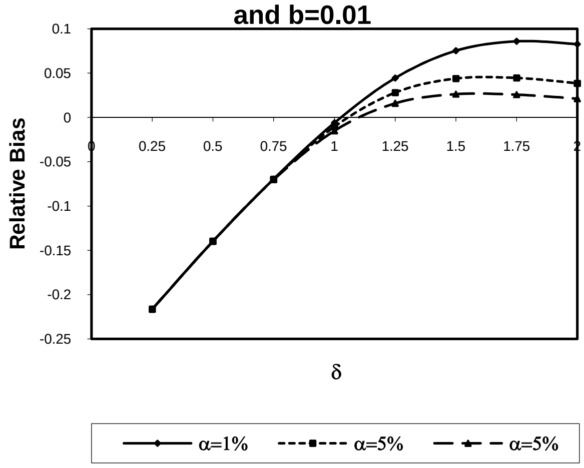

Fig 17: Relative Bias of Preliminary Test Estimator âPTS1 when n=21, d=0.3 and b=0.01

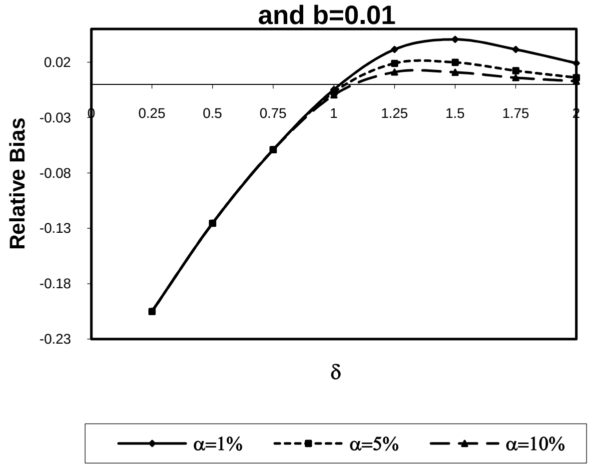

Fig 18: Relative Efficiency of Preliminary Test Estimator aPTS1 for Shape Parameter when n=5, d=0.3 and b=0.01

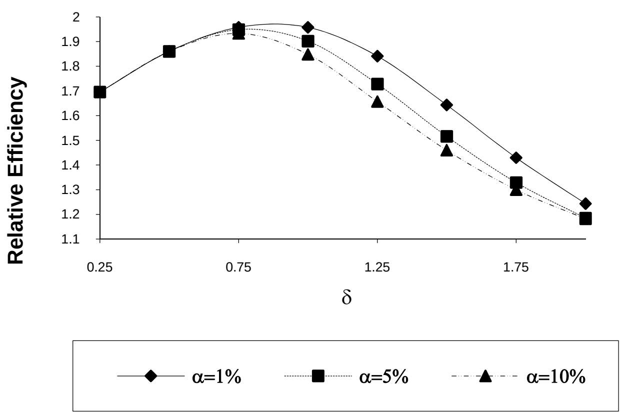

Fig 19: Relative Efficiency of Preliminary Test Estimator aPTS1 for Shape Parameter when n=10, d=0.3 and b=0.01

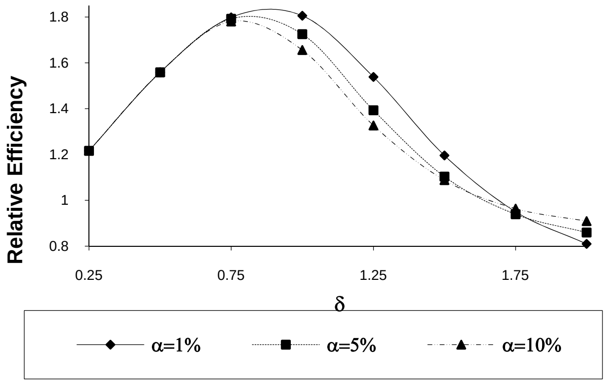

Fig 20: Relative Efficiency of Preliminary Test Estimator aPTS1 for Shape Parameter when n=21, d=0.3 and b=0.01

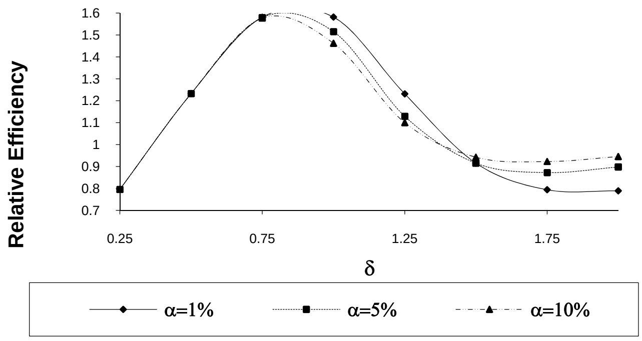

Fig 21: Relative Bias of Preliminary Test Estimator âPTS2 when n=5, d=0.3 and b=0.01

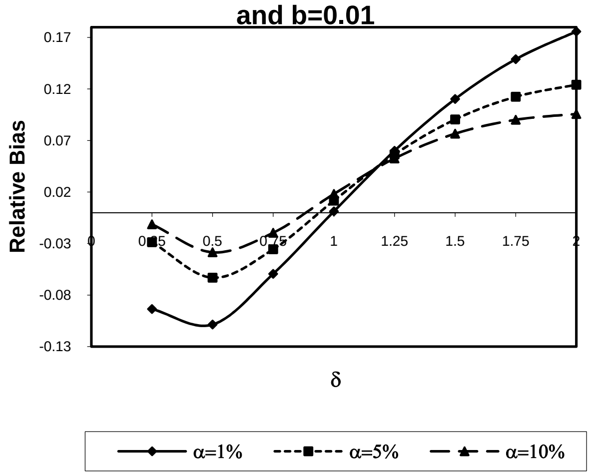

Fig 22: Relative Bias of Preliminary Test Estimator âPTS2 when n=10, d=0.3 and b=0.01

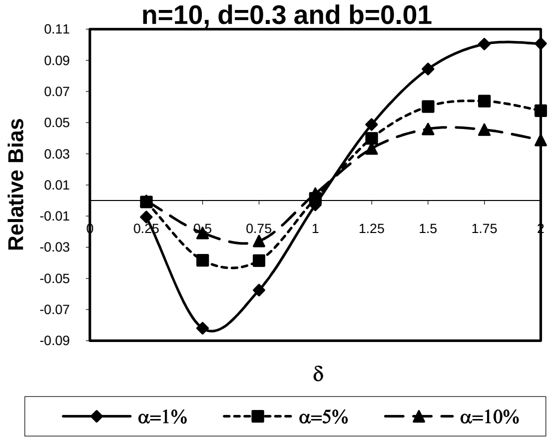

Fig 23: Relative Bias of Preliminary Test Estimator âPTS2 when n=21, d=0.3 and b=0.01

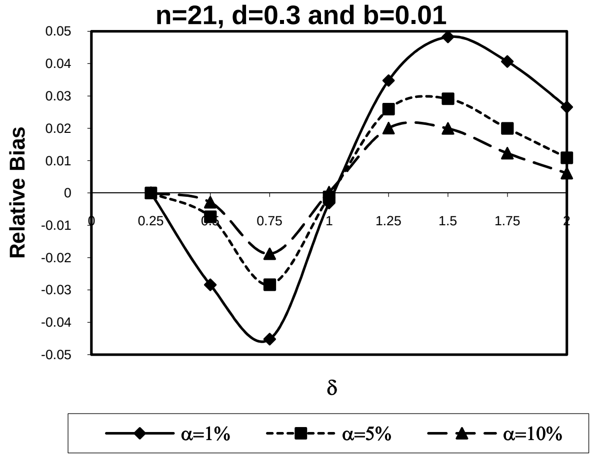

Fig 24: Relative Efficiency of Preliminary Test Estimator âPTS2 for Shape Parameter when n=5, d=0.3 and b=0.01

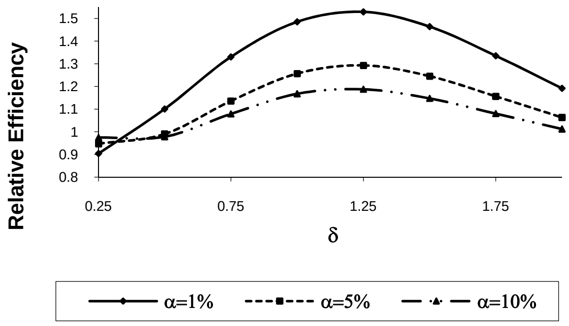

Fig 25: Relative Efficiency of Preliminary Test Estimator âPTS2 for Shape Parameter when n=10, d=0.3 and b=0.01

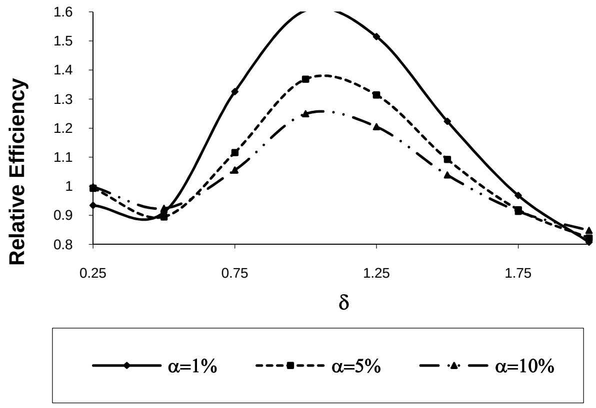

Fig 26: Relative Efficiency of Preliminary Test Estimator âPTS2 for Shape Parameter when n=21, d=0.3 and b=0.01