CHAIN TYPE ESTIMATORS FOR RATIO OF TWO POPULATION MEANS USING AUXILIARY CHARACTERS IN THE PRESENCE OF NON-RESPONSE

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Abstract

This paper presents two chain type estimators for ratio of two population means of study characters using auxiliary characters in the presence of non- response. The expressions for the relative bias and mean square errors of the proposed estimators are derived. The present estimators are more efficient than the relevant estimators for fixed values of first phase sample n^1 and second phase sample n, under the specified conditions. We also find that the proposed estimators are more efficient than the corresponding estimators for $R(=\overline{Y}_1/\overline{Y}_2)$ in case of fixed cost and have less total cost in comparison to the cost of the relevant estimators for specified precision.

1. Introduction

In the problem of estimations of ratio of two population means, we use the ratio of unbiased estimate of each population means. In order to improve the efficiency of the estimator of $R(=\overline{Y}_1/\overline{Y}_2)$, the ratio of two population means, the use of auxiliary character may be made. The estimation of ratio of two population means using an auxiliary character was considered by Singh (1965), Tripathi (1970), Upadhayay *et al.* (2000), Singh (1998) and Birader and Singh (1997-1998). By using the information on additional auxiliary character, Srivastava *et al.* (1988, 1989), Singh *et al.* (1994a, 1994b) and Singh and Singh (1997-1998), have proposed different type estimators for ratio of two population means.

During the sample survey, sometimes, we do not get complete information for all the units selected in the sample due to the problem of non response. Estimation of the

ratio of two population means in sample surveys when some observations are missing due to random non-response has been considered by Toutenberg and Srivastava (1998) and Singh and Tracy (2001). Using the Hansen and Hurwitz (1946) technique of sub-sampling from non-respondents, the estimator for ratio of two population mean have been proposed by Khare and Pandey (2000), Khare and Sinha (2002, 2004, 2007) in different situations.
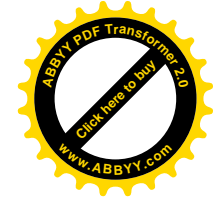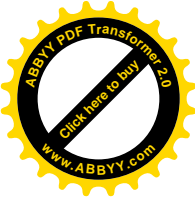
In this paper, we have proposed two chain type estimators for ratio of two population means of the study characters using the auxiliary and additional auxiliary characters in the presence of non-response. The expressions for the relative bias and mean square errors of the proposed estimators are obtained and comparisons of the proposed estimators have been made with the relevant estimators. The optimum values of the size first phase sample (n^1), second phase sample (n) and the sub-sampling fraction $(1/k, \, k>1)$ have been obtained for fixed cost $C \leq D_0$ and for the specified precession $V \leq U_0'$. The performances of the proposed estimators are compared with relevant estimators.

2. The Estimators

Let $Y_{il} : (i = 1,2; l = 1, 2 .... N)$, X_l and Z_l be the non-negative value of lth unit of the population on the study characters y_i $(i=1, \, 2)$, the auxiliary character x and the additional auxiliary character z with their population means $\overline{Y}_i, (i = 1, 2)$, $\overline{X}$ and $\overline{Z}$. In case when $\overline{X}$ is not known, but $\overline{Z}$, the population mean of addition auxiliary character z (closely related to x) is known, it is assumed that additional auxiliary character z may be cheaper and less correlated to the study characters y_1 and y_2 in comparison to the main auxiliary character x. In this case we take a first phase sample of size n^1 from the population of size N by using SRSWOR scheme and we estimate the population mean $\overline{X}$ using additional auxiliary character with known population mean $\overline{Z}$, sample means $\overline{x}^1$ and $\overline{z}^1$ based n^1 observations.

Again a second phase sample of size $n(< n^1)$ is drawn from n^1 first phase sample by SRSWOR method of sampling and it has been observed that n_1 units respond and n_2 units do not respond in the sample of size n for the study characters y_i, $(i = 1, \, 2)$. It is also assumed that the population of size N is composed of N_1 responding and N_2 non-responding units, though they are unknown. Further from n_2 non-responding units, we select a sub-sample of size r $(r = n_2/k, \, k >1)$ using SRSWOR technique of sampling by making extra effort and observe y_i $(i= 1, \, 2)$. Hence, we have $(n_1 + r)$ observations on the y_i $(i=1, \, 2)$ characters. Using the Hansen and Hurwitz (1946) techniques, the estimator for $\overline{Y}_i (i = 1, 2)$ based on (n_1+r) units is given by $(i=1, \, 2)$

$$\overline{y}_i^* = \frac{n_1}{n} \overline{y}_{i(1)} + \frac{n_2}{n} \overline{y}_{i(2)}^{'} \ , \quad i = 1, 2 \, , \tag{2.1}$$

where $\bar{y}_{i(1)}$ and $\bar{y}'_{i(2)}$ are the samples means of characters y_i based on n_1 and r units respectively. The estimator $\bar{y}^*_i$ is unbiased and has variance

$$V(\bar{y}^*_i) = \frac{f}{n} S^2_{yi} + \frac{W_2(k-1)}{n} S^{*\,2}_{yi} , \mathrm{I} = 1, 2 ,$$ (2.2)

where

$f = \dfrac{N-n}{N}, \mathrm{W}_i = \dfrac{N_i}{N}, (i = 1, 2), S^2_{yi}$ and $S^{*\,2}_{yi}$ are the population mean squares of y_i

for the entire population and for the non-responding part of the population.

Similarly the estimator $\bar{x}^*$ for the population mean $\bar{X}$ is given by

$$\bar{x}^* = \frac{n_1}{n} \bar{x}_1 + \frac{n_2}{n} \bar{x}'_2$$ (2.3)

and

$$V(\bar{x}^*) = \frac{f}{n} S^2_x + \frac{W_2(k-1)}{n} S^{*2}_x ,$$ (2.4)

where S^2_x and $S^{*\,2}_x$ are population mean square of x for the entire population and non-responding part of the populations.

Let $\hat{R}(= \dfrac{\bar{y}^*_1}{\bar{y}^*_2})$ denotes a conventional estimator for the ratio of two population means

$R(= \dfrac{\bar{Y}_1}{\bar{Y}_2})$.

In case when $\bar{X}$ is not known, one may estimate it by the sample mean $\bar{x}^1$, based on a preliminary large sample of size $n^1 (> n)$ drawn without replacement. In this situation when we have incomplete information both on the study characters y_i and incomplete /complete information on auxiliary character x from the sample of size n, we may then define the conventional and alternate estimators for the ratio of two population means using an auxiliary character in the presence of non response, which are given as follows

$$T^*_1 = \frac{\bar{y}^*_1}{\bar{y}^*_2} \frac{\bar{x}^*}{\bar{x}^1} , \quad T^*_2 = \frac{\bar{y}^*_1}{\bar{y}^*_2} \frac{\bar{x}}{\bar{x}^1},$$ (2.5)

where $\qquad \bar{x} = \dfrac{1}{n} \sum_{l=1}^{n} x_l , \bar{x}^1 = \dfrac{1}{n^1} \sum_{l=1}^{n^1} x_l$

If $\bar{X}$ is not known, but $\bar{Z}$, the population mean of an additional auxiliary character z (closely related to x) is known. In this situation, we take a first phase sample n^1 from the

population of size N with SRSWOR sampling scheme and estimate the population mean $\overline{X}$ by using the known additional population mean $\overline{Z}$ and the sample means $\overline{x}^1$ and $\overline{z}^1$ based on n^1 observation. We find that $\hat{\overline{X}}_r = \dfrac{\overline{x}^1}{\overline{z}^1}\overline{Z}$ is more precise than sample mean $\overline{x}^1$

if $\rho_{xz} > \dfrac{1}{2}\dfrac{C_z}{C_x}$, where ρ_{xz} is the correlation coefficient between (x, z),

$$C_x = \frac{S_x}{\overline{X}}, C_z = \frac{S_z}{\overline{Z}}, \overline{z}^1 = \frac{1}{n^1}\sum_{l=1}^{n^1} z_l .$$

In case, when we have incomplete information both on the study characters $y_i : i = 1, 2$ and incomplete/complete information on auxiliary character x from the sample of size n, we propose conventional and alternative chain type estimators for ratio of two population means of study characters using auxiliary characters in the presence of non-response, which are given as follows:

$$t_1^* = \frac{\overline{y}_1^*}{\overline{y}_2^*}\frac{\overline{x}^*}{\overline{x}^1}\frac{\overline{z}^1}{\overline{Z}}, \quad t_2^* = \frac{\overline{y}_1^*}{\overline{y}_2^*}\frac{\overline{x}}{\overline{x}^1}\frac{\overline{z}^1}{\overline{Z}} , \tag{2.6}$$

3. Expression for the Relative Bias (RB) and
Mean Square Error (MSE) of the proposed estimators

Using the large sample approximations, the expression for the relative bias and mean square error of the estimators t_1^* and t_2^* up to the terms of order $(1/n)$ are given by

$$RB(t_1^*) = RB(\hat{R}) + \left[\left(\frac{1}{n} - \frac{1}{n^1}\right)A + \frac{W_2(k-1)}{n}A_2 + \frac{f^1}{n^1}B\right],$$
$$\tag{3.1}$$

$$RB(t_2^*) = RB(\hat{R}) + \left[\left(\frac{1}{n} - \frac{1}{n^1}\right)A + \frac{f^1}{n^1}B\right],$$
$$\tag{3.2}$$

$$MSE(t_1^*) = MSE(\hat{R}) + R^2\left[\left(\frac{1}{n} - \frac{1}{n^1}\right)\left(C_x^2 + 2A\right) + \frac{W_2(k-1)}{n}\left(C_x^{*2} + 2A_2\right)\right.$$
$$\left. + \frac{f^1}{n^1}\left(C_z^2 + 2B\right)\right] ,$$
$$\tag{3.3}$$

and

$$MSE(t_2^*) = MSE(\hat{R}) + R^2\left[\left(\frac{1}{n} - \frac{1}{n^1}\right)\left(C_x^2 + 2A\right) + \frac{f^1}{n^1}\left(C_z^2 + 2B\right)\right], \tag{3.4}$$

where

$$RB(\hat{R}) = \frac{f}{n}\left(C_{y2}^2 - C_{y1y2}\right) + \frac{W_2(k-1)}{n}\left(C_{y2}^{*\ 2} - C_{y1y2}^*\right), \tag{3.5}$$

$$MSE(\hat{R}) = R^2\left[\frac{f}{n}\left(C_{y1}^2 + C_{y2}^2 - 2C_{y1y2}\right) + \frac{W_2(k-1)}{n}\left(C_{y1}^{*\ 2} + C_{y2}^{*\ 2} - 2C_{y1y2}^*\right)\right], \tag{3.6}$$

$$A = (C_{y1x} - C_{y2x}),\ A_2 = (C_{y1x}^* - C_{y2x}^*),\ B = (C_{y1z} - C_{y2z}),$$

$$C_{yi} = \frac{S_{yi}}{\overline{Y}_i}, C_{yi}^* = \frac{S_{yi}^*}{\overline{Y}_i}(i = 1, 2), C_x = \frac{S_x}{\overline{X}}, C_{y1y2} = \rho_{y1y2}C_{y1}C_{y2},$$

$$C_{y1y2}^* = \rho_{y1y2}^* C_{y1}^* C_{y2}^*, \qquad C_{y1x} = \rho_{y1x}C_{y1}C_x, C_{y2x} = \rho_{y2x}C_{y2}C_x,$$

$$C_{y1x}^* = \rho_{y1x}^* C_{y1}^* C_x^*, C_{y2x}^* = \rho_{y2x}^* C_{y2}^* C_x^*, C_{y1z} = \rho_{y1z}C_{y1}C_z, C_{y2z} = \rho_{y2z}C_{y2}C_z,$$

$$C_x^* = \frac{S_x^*}{\overline{X}}, C_z = \frac{S_z}{\overline{Z}}, f^1 = \left(1 - \frac{n^1}{N}\right), S_{yi}^2 = \frac{1}{N-1}\sum_{l=1}^{N}(Y_{il} - \overline{Y}_i)^2,$$

$$S_{yi}^{*\ 2} = \frac{1}{N_2 - 1}\sum_{l=1}^{N_2}(Y_{il(2)} - \overline{Y}_{i(2)})^2, S_x^2 = \frac{1}{N-1}\sum_{l=1}^{N}(X_l - \overline{X})^2,$$

$$S_x^* = \frac{1}{N_2 - 1}\sum_{l=1}^{N_2}(X_{l(2)} - \overline{X}_2)^2, S_z^2 = \frac{1}{N-1}\sum_{l=1}^{N}(Z_l - \overline{Z})^2,$$

ρ_{y1y2} and ρ_{y1y2}^* are the correlation coefficients between (y_1, y_2), ρ_{yix} and ρ_{yix}^* are the correlation coefficients between (y_i, x) for entire and non- responding part of the population respectively and ρ_{yiz} is the correlation coefficients between (y_i, z); $i=1, 2$.

Relative bias and mean square error of estimators T_1^* and T_2^*

$$RB(T_1^*) = RB(\hat{R}) + \left(\frac{1}{n} - \frac{1}{n^1}\right)A + \frac{W_2(k-1)}{n}A_2,$$

$$\tag{3.7}$$

$$RB(T_2^*) = RB(\hat{R}) + \left(\frac{1}{n} - \frac{1}{n^1}\right)A, \tag{3.8}$$

$$MSE(T_1^*) = MSE(\hat{R}) + R^2\left[\left(\frac{1}{n} - \frac{1}{n^1}\right)\left(C_x^2 + 2A\right) + \frac{W_2(k-1)}{n}\left(C_x^{*2} + 2A_2\right)\right], \tag{3.9}$$

and

$$MSE(T_2^*) = MSE(\hat{R}) + R^2\left[\left(\frac{1}{n} - \frac{1}{n^1}\right)\left(C_x^2 + 2A\right)\right], \tag{3.10}$$

4. Comparison of Mean Square Errors of t_1^* and t_2^* with Respect to T_1^*, T_2^* and $\hat{R}$

$$MSE(t_1^*) < MSE(\hat{R}) \text{ If,} \tag{4.1}$$

$$\left(\rho_{y1x}\frac{C_{y1}}{C_x} - \rho_{y2x}\frac{C_{y2}}{C_x}\right) < -\frac{1}{2}, \quad \left(\rho_{y1x}^*\frac{C_{y1}^*}{C_x^*} - \rho_{y2x}^*\frac{C_{y2}^*}{C_x^*}\right) < -\frac{1}{2}$$

$$\text{and } \left(\rho_{y1z}\frac{C_{y1}}{C_z} - \rho_{y2z}\frac{C_{y2}}{C_z}\right) < -\frac{1}{2}.$$
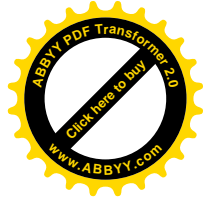
$$MSE(t_1^*) < MSE(T_1^*) \text{ If,} \tag{4.2}$$

$$\left(\rho_{y1z}\frac{C_{y1}}{C_z} - \rho_{y2z}\frac{C_{y2}}{C_z}\right) < -\frac{1}{2}.$$

$$MSE(t_2^*) < MSE(\hat{R}) \text{ If,} \tag{4.3}$$

$$\left(\rho_{y1x}\frac{C_{y1}}{C_x} - \rho_{y2x}\frac{C_{y2}}{C_x}\right) < -\frac{1}{2} \text{ and } \left(\rho_{y1z}\frac{C_{y1}}{C_z} - \rho_{y2z}\frac{C_{y2}}{C_z}\right) < -\frac{1}{2}.$$

$$MSE(t_2^*) < MSE(T_2^*) \text{ If,} \tag{4.4}$$

$$\left(\rho_{y1z}\frac{C_{y1}}{C_z} - \rho_{y2z}\frac{C_{y2}}{C_z}\right) < -\frac{1}{2}.$$

5. Determination of n^1, n and k for the Fixed Cost $C \le D_0$

Let D_0 be the total cost (fixed) of the survey apart from overhead cost. The expected total cost of the survey apart from overhead cost is given by

$$C = (c_1^{'} + c_2^{'})n^1 + n(c_1 + c_2 W_1 + c_3 \frac{W_2}{k}) \ , \tag{5.1}$$

where

$c_1^{'}$ – the cost per unit of identifying and observing auxiliary character x,

$c_2^{'}$ – the cost per unit of obtaining information on auxiliary character z,

c_1 – the cost per unit of mailing questionnaire visiting the unit at the second phase,

c_2 – the cost per unit of collecting and processing data for the study characters y_1 and y_2 obtained from n_1 responding units and

c_3 – the cost per unit of obtaining and processing data for the study characters y_1 and y_2 (after extra effort) from the sub sampled units.

The $MSE(t_i^*), i = 1, 2$ can be expressed in terms of the notation $R^2 U_{0i}$, $R^2 U_{1i}$ and $R^2 U_{2i}$ which is given as

$$MSE(t_i^*) = \left[\frac{1}{n} R^2 U_{0i} + \frac{1}{n^1} R^2 U_{1i} + \frac{k}{n} R^2 U_{2i} \right]$$
$$+ \text{ Independent terms of } n, \ n^1 \text{ and } k \ , \tag{5.2}$$

where $R^2 U_{0i}$, $R^2 U_{1i}$ and $R^2 U_{2i}$ are respectively the coefficients of terms of $\frac{1}{n}, \frac{1}{n^1}$ and $\frac{k}{n}$ in the expressions of $MSE(t_i^*), i = 1, 2$.

Let us define a function ϕ^* for minimizing the $MSE(t_i^*)$ and to obtain the optimum values of n, n^1 and k in the case of fixed cost $C \le D_0$ which is given as

$$\phi^* = MSE(t_i^*) + \lambda_i \left\{ (c_1^{'} + c_2^{'})n^1 + n\left(c_1 + c_2 W_1 + c_3 \frac{W_2}{k} \right) \right\}, \tag{5.3}$$

where λ_i is Lagrange's multiplier.

Now differentiating ϕ^* with respect to n, n^1 and k and equating to zero, we have

$$n^1 = R \sqrt{\frac{U_{1i}}{\lambda_i (c_1^{'} + c_2^{'})}} \ , \tag{5.4}$$

$$n = R \sqrt{\frac{(U_{0i} + kU_{2i})}{\lambda_i \left(c_1 + c_2 W_1 + c_3 \dfrac{W_2}{k} \right)}} \tag{5.5}$$

and

$$\frac{n}{k} = R \sqrt{\frac{U_{2i}}{\lambda_i c_3 W_2}} \quad . \tag{5.6}$$

By putting the value of n from (5.5) in (5.6) and after solving, we get

$$k_{opt} = \sqrt{\frac{c_3 W_2 U_{0i}}{(c_1 + c_2 W_1) U_{2i}}} \quad . \tag{5.7}$$

Now, putting the values of n^1 and n from (5.4) and (5.5), using the value of k_{opt} from (5.7) in (5.1), we have

$$\sqrt{\lambda_i} = \frac{R}{D_0} \left[\sqrt{U_{1i}(c_1^{'} + c_2^{'})} + \sqrt{(U_{0i} + k_{opt} U_{2i}) \left(c_1 + c_2 W_1 + c_3 \frac{W_2}{k_{opt}} \right)} \right] \quad . \tag{5.8}$$

It has been observed that the determinant of the matrix of second order derivative of ϕ^* with respect to n^1, n and k is positive for the optimum values of n^1, n and k, which shows that the solutions for n^1, n given by (5.4) and (5.5), using (5.7) and (5.8), and the optimum value of k under the condition $C \leq D_0$ minimize the mean square error of t_i^*.
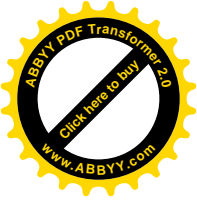
Hence for the optimum values of n^1, n and k, the minimum mean square error of t_i^* is given by

$$MSE(t_i^*)_{\min} = R^2 \left[\frac{1}{D_0} \left\{ \sqrt{U_{1i}(c_1^{'} + c_2^{'})} + \sqrt{(U_{0i} + k_{opt} U_{2i}) \left(c_1 + c_2 W_1 + c_3 \frac{W_2}{k_{opt}} \right)} \right\}^2 \right.$$
$$\left. - \frac{(C_{y1}^2 + C_{y2}^2 - 2\rho_{y1y2} C_{y1} C_{y2} + C_z^2 + 2B)}{N} \right] \quad . \tag{5.9}$$

Now neglecting the term of order ($1/N$), we have

$$MSE(t_i^*)_{\min} = R^2 \left[\frac{1}{D_0} \left\{ \sqrt{U_{1i}(c_1^{'} + c_2^{'})} + \sqrt{(U_{0i} + k_{opt} U_{2i}) \left(c_1 + c_2 W_1 + c_3 \frac{W_2}{k_{opt}} \right)} \right\} \right]^2 \quad . \tag{5.10}$$

6. Determination of n^1, n and k for the Specified Precision $V = U_0^{'}$

Let $U_0^{'}$ be the specified variance of the estimators t_i^{*} fixed in advance. So we have

$$U_0^{'} = R^2 \left(\frac{U_{0i}}{n} + \frac{U_{1i}}{n^1} + \frac{kU_{2i}}{n} \right) - R^2 \frac{(C_{y1}^2 + C_{y2}^2 - 2\rho_{y1y2}C_{y1}C_{y2} + C_z^2 + 2B)}{N}. \qquad (6.1)$$

For minimizing the average total cost C for the specified variance (i.e. $MSE(t_i^{*}) = U_0^{'}$; $i=1, 2$) of the estimators t_i^{*} and for obtaining the optimum values of n^1, n and k, we define a function ψ^{*} given by

$$\psi^{*} = \left\{ \left(c_1^{'} + c_2^{'} \right) n^1 + n \left(c_1 + c_2 W_1 + c_3 \frac{W_2}{k} \right) \right\} - \mu_i \left(MSE(t_i^{*}) - U_0^{'} \right), \qquad (6.2)$$

where μ_i is Lagrange's multiplier.

Now, by differentiating ψ^{*} with respect to n^1, n, k and equating to zero, we get

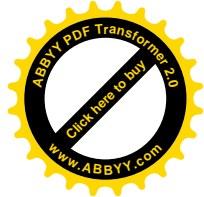$$n^1 = R \sqrt{\frac{\mu_i U_{1i}}{(c_1^{'} + c_2^{'})}}, \qquad (6.3)$$

$$n = R \sqrt{\frac{\mu_i (U_{0i} + kU_{2i})}{\left(c_1 + c_2 W_1 + c_3 \frac{W_2}{k} \right)}}, \qquad (6.4)$$

and

$$k_{opt} = \sqrt{\frac{U_{0i} c_3 W_2}{U_{2i}(c_1 + c_2 W_1)}}. \qquad (6.5)$$

The value of μ_i is given by

$$\sqrt{\mu_i} = R \frac{\left[\sqrt{(c_1^{'} + c_2^{'})U_{1i}} + \sqrt{\left(U_{0i} + k_{opt}U_{2i}\right)\left(c_1 + c_2 W_1 + c_3 \frac{W_2}{k_{opt}} \right)} \right]}{\left[U_0^{'} + \frac{R^2(C_{y1}^2 + C_{y2}^2 - 2\rho_{y1y2}C_{y1}C_{y2} + C_z^2 + 2B)}{N} \right]}. \qquad (6.6)$$

The optimum values of n^1 and n have been obtained by putting the value of k_{opt} and the optimum value $\sqrt{\mu_i}$ from (6.5) and (6.6) in (6.3) and (6.4) respectively. Further we observe that the determinant of the matrix of the second order derivative of ψ^* with respect to n^1, n and k is positive for the optimum values of n^1, n and k. The minimum expected total cost to be incurred on the use of t_i^* for the specified precision will be given by

$$C(t_i^*)_{min} = \frac{R^2\left[\sqrt{(c_1^{'}+c_2^{'})U_{1i}} + \sqrt{(U_{0i}+k_{opt}U_{2i})\left(c_1+c_2W_1+c_3\dfrac{W_2}{k_{opt}}\right)}\,\right]^2}{\left[U_0^{'} + \dfrac{R^2(C_{y1}^2+C_{y2}^2-2\rho_{y1y2}C_{y1}C_{y2}+C_z^2+2B)}{N}\right]}. \quad (6.7)$$

Now neglecting the term of order (1/N), we have

$$C(t_i^*)_{min} = \frac{R^2\left[\sqrt{(c_1^{'}+c_2^{'})U_{1i}} + \sqrt{(U_{0i}+k_{opt}U_{2i})\left(c_1+c_2W_1+c_3\dfrac{W_2}{k_{opt}}\right)}\,\right]^2}{U_0^{'}}. \quad (6.8)$$

7. An Empirical Study

The present data belong to the data on physical growth of upper socio- economic group of 95 school going children of Varanasi under an ICMR study, Department of Pediatrics, BHU during 1983-84 has been taken under study,(Khare and Sinha (2007)). The first 25% (i.e. 24 children) units have been considered as non-response units. The values of parameters related to the study characters y_1 (the height of children in cm.) and y_2 (weight of children in kg), the auxiliary character x (chest circumference of the children in cm) and additional auxiliary character z (skull circumference of children in cm) have been given as follows:
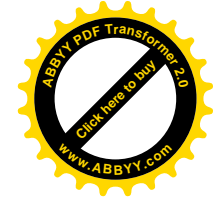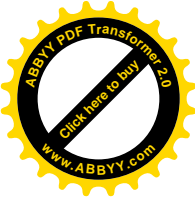
$$\overline{Y}_1 = 115.9526, \quad \overline{Y}_2 = 19.4968, \quad \overline{X} = 55.8611, \quad \overline{Z} = 51.1726,$$

$$C_{y1} = 0.05146, \quad C_{y2} = 0.15613, \quad C_x = 0.05860, \quad C_z = 0.03006,$$

$$C_{y1}^* = 0.04402, \quad C_{y2}^* = 0.12075, \quad C_x^* = 0.05402, \quad \rho_{y1x} = 0.620,$$

$$\rho_{y2x} = 0.846, \quad \rho_{y1z} = 0.374, \quad \rho_{y2z} = 0.328, \quad \rho_{y1x}^* = 0.401,$$

$$\rho_{y2x}^* = 0.729, \quad \rho_{y1y2} = 0.713, \quad \rho_{y1y2}^* = 0.678.$$

The problem considered is to estimate the ratio between height and weight of the male children aged 6-7 years using chest circumference as the auxiliary character and skull circumference as the additional auxiliary character.

Table 1

Mean square error (MSE) and Relative efficiency (RE) of the estimators t_1^*, t_2^*, T_1^* and T_2^* with respect to $\hat{R}$ for $k=2$, 3, 4 ($N=95$, $n^1=70$, $n=35$).
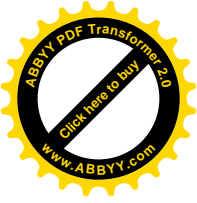
Estimators	1/k		
	1/4	1/3	1/2
$\hat{R}$	100 (0.01707)*	100 (0.01469)	100 (0.01231)
T_1^*	184 (0.00928)	181 (0.00810)	178 (0.00692)
T_2^*	133 (0.01287)	140 (0.01049)	152 (0.00812)
t_1^*	187 (0.00915)	184 (0.00797)	181 (0.00678)
t_2^*	134 (0.01273)	142 (0.01036)	154 (0.00798)

*Figures in parenthesis give the MSE (.).

Table 2

Relative efficiency (RE) of t_1^*, t_2^*, T_1^* and T_2^* with respect to $\hat{R}$ for fixed cost $C_0=Rs.225$: $(c_1^{'} = Rs\ 0.95, c_2^{'} = Rs\ 0.10,\ c_1 = Rs\ 2, c_2 = Rs\ 5, c_3 = Rs\ 15)$.

Estimators	k_{opt}	n_{opt}^1	n_{opt}	R. E (.)
$\hat{R}$	1.93	-----	29	100 (0.7608)
T_1^*	1.86	61	21	117 (0.6516)
T_2^*	1.17	60	19	111 (0.6852)
t_1^*	1.86	55	22	118 (0.6464)
t_2^*	1.17	54	20	112 (0.6799)

*Figures in parenthesis give the MSE (.).

Table 3

Expected cost of the estimators $\hat{R}$, T_1^*, T_2^*, t_1^* and t_2^* for the specified precision

$U_0^{'}=0.674$: $(c_1^{'} = Rs\ 0.95, c_2^{'} = Rs\ 0.10,\ c_1 = Rs\ 2,\ c_2 = Rs\ 5, c_3 = Rs\ 15)$.

Estimators	k_{opt}	n_{opt}^1	n_{opt}	Expected cost in (Rs.)
$\hat{R}$	1.93	-----	33	252
T_1^*	1.86	59	21	216
T_2^*	1.17	60	19	227
t_1^*	1.86	52	20	214
t_2^*	1.17	54	19	225

From table-1, we observed that for fixed values of n^1 and n, the estimators' t_1^* and t_2^* are more efficient than $\hat{R}$ and corresponding estimators T_1^* and T_2^* for different values of k. The estimators T_1^* and T_2^* are more efficient than $\hat{R}$. It has also been observed that the estimator t_1^* is more efficient to the estimator t_2^*.

From table-2, we observe that for fixed cost, the estimators t_1^* and t_2^* have smaller mean square error than that of $\hat{R}$ and t_1^* and t_2^* are also more efficient than the corresponding estimators T_1^* and T_2^*.

From table-3, we observed that the expected costs incurred in t_1^* and t_2^* are less in comparison to the expected cost incurred for $\hat{R}$, T_1^* and T_2^* in the case of specified precision.

8. Conclusion

The proposed chain type estimators t_1^* and t_2^* for ratio of two population means using an additional auxiliary character , it is a extension of the earlier proposed estimators for $\hat{R}$ using one auxiliary character in the presence of non-response.

The empirical study shows that the use of additional auxiliary character in the proposed estimators helps:

(i)   In decreasing the mean square errors of t_1^* and t_2^* in comparisons to the relevant estimators for the fixed $\left((n^1, n)\right)$ and also for the fixed cost D_0 .

(ii) In decreasing the cost of t_1^* and t_2^* in comparisons to the relevant estimators in the case of specified precision.

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