

Study of Limb Darkening Effect and Rotation Period of Sun by using Solar Telescope

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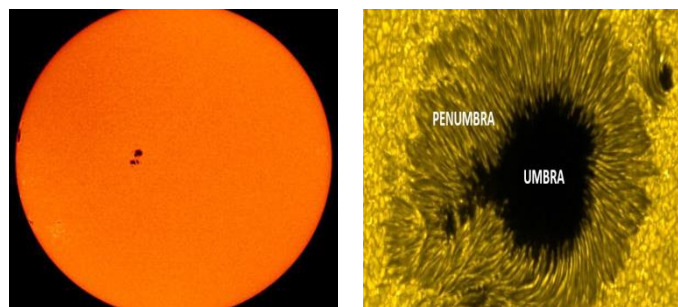
Abstract: Dynamic Sun that directly or indirectly controls the whole geospace environment and affects the space-borne (satellites, rockets) and ground based technological systems (communication, navigation). Numbers of sunspots seen on the solar surface are known as the measure of solar activity. By tracking these sunspots we have calculated solar rotational time period (synodic and sidereal time period). To that end, we have used the unique facility of a solar telescope, a CCD camera and a solar spectrometer installed at Physics Department, Banaras Hindu University, Varanasi. We observed full solar disc images during morning hours (9:15 am to 9:30 am) from April, 8-18, 2019. The synodic time period and the sidereal time period were calculated as 26.94 days and 25.09 days respectively. It is also observed that the solar disc intensity also known as solar limb darkening effect decreases from center to the limb. To study the solar limb darkening effect, we have taken full solar disk images free from sunspots during morning hours from 28th March, 2019 to 8th April, 2019 and analyzed their intensity variations to calculate solar limb darkening coefficient of the Sun. The average solar limb darkening coefficient comes out to be 0.61. Our above observed results are found to be in consistent with the other reported results.

Index Terms: Solar Limb Darkening Effect, Synodic time period, Sidereal time period, Solar Telescope, Sunspots.

I. INTRODUCTION

The study of astronomy and astrophysics became interesting after the discovery of first astronomical telescope by Galileo Galilei (Kitchin, 1995). Our sun is a typical star of Milky Way galaxy. By using a specific telescope with solar filter we can see its upper surface very clearly. In ancient time people treat Sun like a God and they think there is no change in Sun. But after the invention of telescope in 1605 AD, Galileo observed that there are some dark regions on surface of sun known as sunspots. Further in 1612 he explained that these sunspots rotate with sun surface, and number of sunspots varies with time (Vaquero and Vazquez, 2009). The number of sunspots

seen on the solar disc represents the strength of the solar activities. The numbers of sunspots varies with time and have cyclic variation which is known as solar cycle with a period of 11 years (Kiepenheuer, 1953, Li et al, 2002). Currently, 24th solar cycle has ended and a new solar cycle 25th has just started showing solar minima period. Thus we observed very less number of sunspots on sun's surface during our analysis period. The center of sunspot is darker and cooler than surrounding called as umbra and its boundary part is brighter and hotter than center called as penumbra (Polygiannakis et al., 1996) as shown in fig. 1. Sun rotates on its axis from west to east and the solar rotation period varies from equator to poles. Since the life time of a sunspot is approximately some days to weeks, it is used to find the rotation period of sun. The solar rotation period is about 25 days at equator and 35 days at poles (Howard, 1978, Kosovichev and Rozelot, 2018). Recently Oghrapishvili et al, (2018) use SDO/AIA data to find out solar rotation period.



(a) Sun image with sunspots. (b) Close image of a sunspot.

Fig. 1. Image of Sun and Sunspot (credit NASA, USA).

Sun produces tremendous energy of radiation produced by nuclear fusion reaction at its center, so it has more temperature at its center and less at its surface. Similarly the density of plasma could high at center than its surface. Due to this phenomenon the brightness of sun decreases from center to edge (or limb). This is known as Limb Darkening Effect (Pierce et al.,

1977). Frank Very (1902) studied the absorption of solar atmosphere and find that at different frequency the limb darkening coefficient varies. It appears more for higher frequency (or low wavelength) (Pierce & Slaughter, 1977). There are two basic reasons for limb darkening effect (Neckel and Labs, 1994):

- (a) The density of star decreases as distance increases from center.
- (b) The temperature of star decreases as distance increases from center.

This can also be understood by the concept of opacity and optical depth. As we see radially at center, we can probe sun more deep as compare to its limb due to its atmospheric thickness at limb (Zeilik and Gregory, 1997). This can be shown in fig. 2. Here L is optical depth, THI is high temperature surface and TLO is low temperature surface. Ramanathan (1954) have reported the limb darkening coefficient equal to 0.6. Recently Moon et al. (2017) have studied in details about the solar limb darkening effect using SOHO and SDO stalactite data and reported the limb darkening coefficient varies from 0.6 to 0.7.

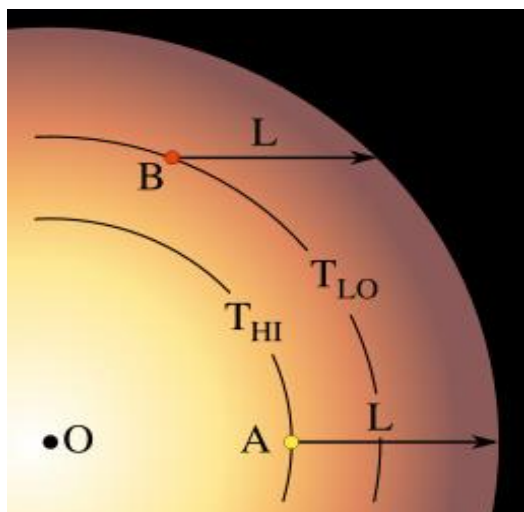


Fig. 2. Limb Darkening Effect (Credit: Zeilik and Gregory, 1997).

The study of limb darkening is very important for a star to define its atmospheric gaseous composition and metal element at its surface.

In the present study, we used solar telescope installed at Banaras Hindu University, Varanasi as a unique facility for observation and measurements of solar parameters. The solar telescope is fitted with an intensity filter to observe solar disk, and records the images and intensity to calculate solar rotation period and limb darkening coefficient. Instrumentation and method of data analysis are presented in section II and results and discussions are given in section III. Finally the conclusions of the study are presented in section IV.

II. INSTRUMENTATION AND METHOD OF ANALYSIS

In this study, our main requirement was to take a clear, high resolution picture of sun. For this prospective we used a telescope with primary lens diameter 90 mm and focal length 1000 mm. Along with this, a CCD camera with three color (red, green, blue) filter inside it (Fig. 3). This camera has 6.1 MP resolutions (3032 × 2016) with 7.8 μm × 7.8 μm pixel size. We are able to change its exposure time from 1 ms to 1 hour. For reducing intensity of the sun we use a solar filter made of Nickel-Chromium which reduces 99.999% solar intensity. A schematic of experimental facility is shown in fig. 3.



Fig. 3. Experimental facilities for solar measurements: (a) Telescope, (b) Solar Filter and (c) CCD camera.

After taking images of full solar disk, we can easily track the sunspots and can find the intensity variation with pixel distribution. These are briefly defined below:

A. Tracking of sunspot

We know that earth is tilted by 23.5 degree from sun's equator (Fig. 4). For removing error due to this condition we must take the image at same time every day. Here we selected time duration between 9:15 to 9:30 hrs IST on each day and taken the images. We observed a single sunspot on west limb of sun on 8 April 2019 and taken the images using CCD camera each day for 11 days on same time until it disappeared in the east limb on 18th April, 2019. As we see the sun from earth and assume that earth is stationary, If we find a sunspot at a fixed latitude it moves θ degree in time 't' then its angular velocity will be:

$$\omega = \frac{\theta}{t} \quad (1)$$

where ω is the angular velocity of the sunspots. Then the value of solar rotation period is known as synodic period of rotation which is given as:

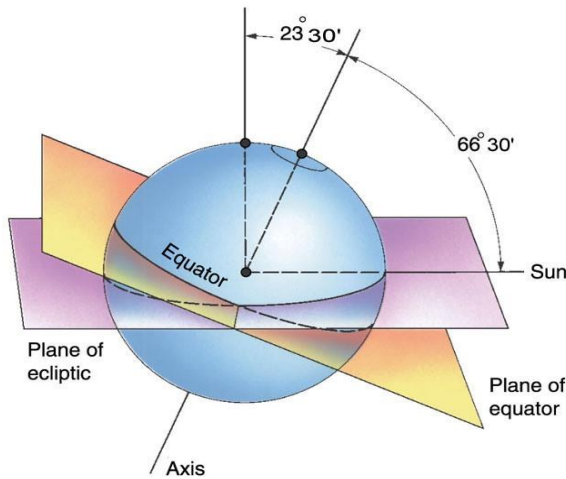


Fig. 4. Inclination of earth from sun's equator. (Source: <http://faculty.kutztown.edu/courtney/blackboard/Physical/21Seasons/seas.html>)

$$S = \frac{360^\circ}{\omega} \quad (2)$$

Since earth is in motion around the sun, the above rotation period is the apparent rotation period of the Sun as seen from the Earth. Hence the actual rotation period of the Sun is the time which takes for a point on the Sun to rotate once with respect to the distant stars which is known as the sidereal period of rotation (Beck, 1999). Since the earth move 360° in 365.25 days in same directions around sun as the sunspots move, the synodic period is a bit longer than the sidereal period. If **P** is the sidereal period of rotation in days, and **S** is the synodic period of rotation of the Sun in days, the relation between **P** and **S** is given as:

$$\frac{360}{P} = \frac{360}{s} + \frac{360}{365.25} \quad (3)$$

$$P = \frac{(S \times 365.25)}{(S + 365.25)} \quad (4)$$

So with the help of the sidereal time period we can measure exact rotation period of sun with the help of the location of the sunspots.

B. Limb Darkening Effect

As we know that Sun is not equally brighter all over disk, its brightness decreases towards limb. The brightness is function of distance from center. This can be understood using Fig. 5 and equation (5). Where *a* is the radius of the solar disc, *r* is radial distance from the centre of the disc. Intensity relation can be written as (Moon et al., 2017):

$$I(\theta) = I(0)[1 - u(1 - \cos\theta)] = I(0)[1 - u(1 - \mu)] \quad (5)$$

here, *u* is the limb darkening coefficient, and $\mu = \cos \theta$.

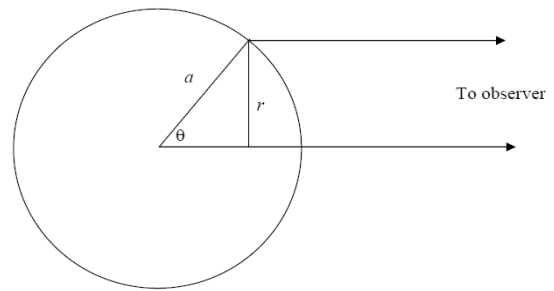


Fig. 5. Observation for limb darkening effect.

In the above equation *I*(0) is the specific intensity at the centre of the disc. If *r* = *a* or $\theta = 90^\circ$ then the specific intensity at limb is *I*(0)(1-*u*). The limb darkening coefficient can be written as:

$$u = \frac{[I(\text{centre}) - I(\text{limb})]}{I(\text{centre})} \quad (6)$$

III. RESULTS AND DISCUSSIONS

A. Rotation Period of Sun

During our measurement we observed a single sunspot on 8 April 2019 which disappeared at 18 April 2019. The positions of the sunspot at different days are shown in fig. 6. The positions (latitudes and longitudes) of sunspots from 8 April 2019 to 18 April 2019 have been tabulated in the table 1 below.

Table 1: Positions of sunspots from 8 April 2019 to 18 April 2019.

Serial number	Days (date)	Time (IST)	Longitudes (degree)	Latitudes (degree)
1	8/04/19	9:27	-67.50	13.50
2	9/04/19	9:22	-56.25	12.75
3	10/04/19	9:29	-42.19	14.25
4	11/04/19	9:20	-29.21	15.25
5	12/04/19	9:25	-15.79	13.50
6	13/04/19	9:17	-2.25	11.25
7	14/04/19	9:18	11.25	9.00
8	15/04/19	9:18	24.47	8.25
9	16/04/19	9:18	39.38	4.50
10	17/04/19	9:20	----*	----*
10	18/04/19	9:21	64.50	3.50

*Note: Data is not available due to cloudy condition.

By the above Table 1 we observed that this sunspot always lies in Northern hemisphere of sun having its latitude varies from 3.50° to 13.50° and longitude varies from -67.50° to $+64.50^\circ$. We have measured the position of sunspot approximately same time in the morning of each day. We have plotted the graph between the positions of sunspot at different longitudes with respect to different dates by using Mathematica software which has been shown in the fig. 7.

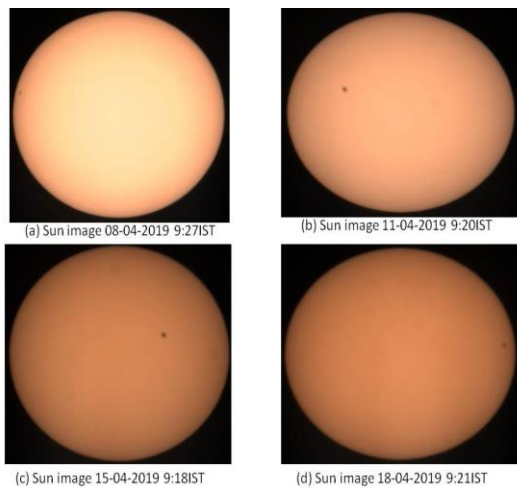


Fig.6. Different positions of sunspot from 8-18 April 2019.

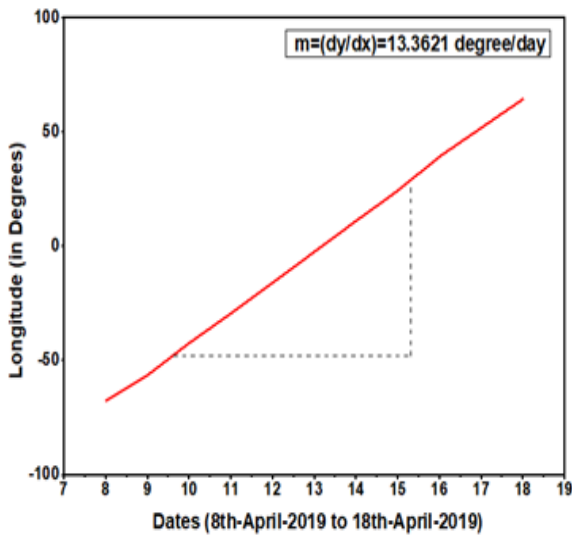


Fig.7. The variation in longitudes of sunspot at different days (8 April 2019-18 April 2019)

From the above graph we have calculated the slope which comes out to be 13.36 degree/days. Thus we find the synodic time period which comes out to be 26.94 days and with the help of the synodic time period we have calculated the sidereal time period which comes out to be 25.09 days. Chen et al., (2015) have also computed the sidereal rotation period of 25 days around equator.

A. *Limb Darkening Coefficient:*

With the help of the CCD camera along with the Image-J software we have analyzed the recorded data and observed the limb darkening effect which is shown in the fig. 8 and fig. 9. From the fig. 8 we can clearly observe the darkening in the intensity towards limb. The fig. 9 shows that the intensity at the one limb of a sun's diameter is very low while in the centre the intensity is very high and it decreases again as we goes towards the another limb.

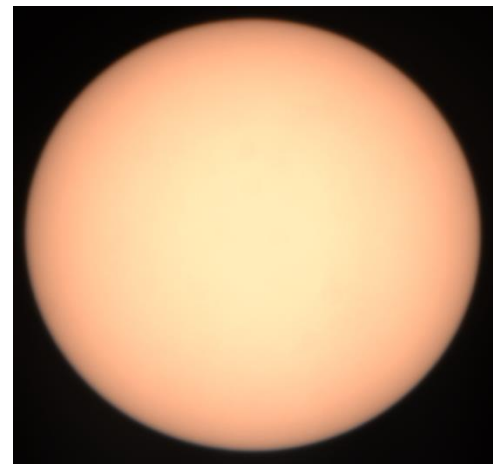


Fig.8. Sun image taken by our telescope along with CCD camera on 04-04-2019 at 9:27 hrs IST.

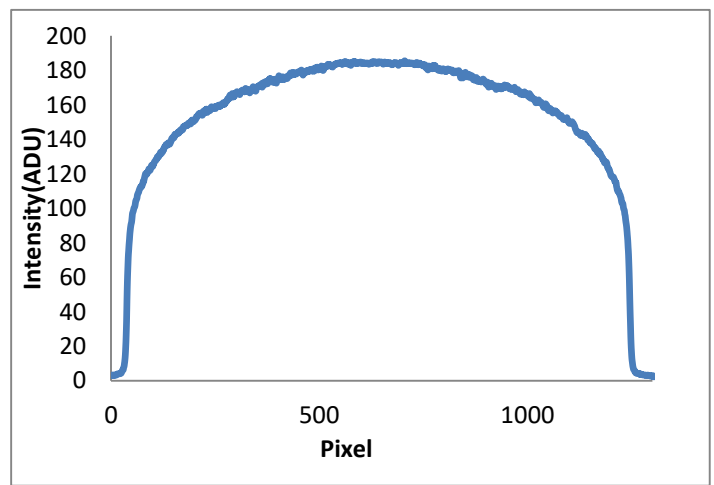


Fig. 9. The Intensity variations from one limb to another limb of a sun's diameter on 04-04-2019 at 9:27 hrs IST.

We observed the sun in the morning (in between 9:00 to 10:00 hrs IST) as well as in evening (in between 16:00 to 17:00 hrs IST) each day. With the help of the observed intensity data and by using equation (6) we have calculated the limb darkening coefficient which came out to be a nearly constant for both morning and as well as in evening period as shown in Table 2.

Table 2: Limb Darkening Coefficient for different days (morning as well as evening hours).

Serial number	Date	Limb Darkening coefficient morning data (u)	Limb Darkening coefficient evening data (u)
1.	28/03/2019	0.60	0.61
2.	29/03/2019	0.62	0.59
3.	30/03/2019	0.62	0.61
4.	01/04/2019	0.60	-----*
5.	02/04/2019	0.60	0.61
6.	04/04/2019	0.60	0.61
7.	05/04/2019	0.61	0.59
8.	06/04/2019	0.61	0.59
9.	08/04/2019	0.61	0.60

Note: *: Data is not available due to cloudy condition.

With the help of the intensity data in the table 2 we observed that edge of sun is averagely 41% less bright than its center. We found the average limb darkening coefficient as 0.61. Our computed limb darkening coefficient is in good agreement with the other reported results which varies from 0.6 to 0.7 (Ramanathan, 1954; Moon et al., 2017).

Thus our specific equipment of solar telescope along with CCD camera and solar spectrometer installed at BHU, Varanasi is highly appropriate for educational purpose in graduate as well as post graduate laboratories/teaching. In this way, reliable results can be easily obtained using a simple methodology.

CONCLUSION

We have used the unique facility of a solar telescope installed at BHU, Varanasi to study different parameters of Sun mainly solar rotation period and solar limb darkening effect. The conclusions of the study are summarized below:

- To compute the solar rotation period we have captured the sunspot images on different days. The computed synodic time period comes out to be 26.94 days and the sidereal time period comes out to be 25.09 days.
- To study the solar limb darkening effect we captured full solar disk images free from sunspots during morning hours from 28th March, 2019 to 8th April, 2019, and analyzed intensity variation to calculate solar limb darkening coefficient of Sun. The average solar limb darkening coefficient comes out to be 0.61.
- Our above computed solar rotation periods and limb darkening coefficients are found to be in a good agreement with the other reported results.

The knowledge of limb darkening effect is important to determine the angular diameter of any stellar object. For construction and verification of solar modal atmosphere limb darkening effect is an important parameter. The limb darkening effects at various wavelengths can give the information of chemical composition in atmosphere of a star.

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