Volume 64, Issue 2, 2020

Journal of Scientific Research

Institute of Science,

Banaras Hindu University, Varanasi, India.

Zero-truncated Poisson-Ishita Distribution and Its Applications

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Abstract: **A zero-truncation of Poisson-Ishita distribution (PID) of Shukla and Shanker (2019) named 'zero-truncated Poisson-Ishita distribution (ZTPID) has been proposed. Moment based measure including coefficient of variation, skewness, kurtosis, and the index of dispersion of the distribution have been presented. The method of maximum likelihood and the method of moments have also been discussed for estimating its parameter. Proposed distribution is applied on two real data sets to test its goodness of fit over zero-truncated Poisson distribution (ZTPD), zero-truncated Poisson-Lindley distribution (ZTPLD), zero-truncated Poisson-Akash distribution (ZTPAD) and zero-truncated Poisson-Sujatha distribution (ZTPSD).**

Index Terms: **Zero-truncated distribution, Poisson-Ishita distribution, Moments, Mathematical and statistical properties, Estimation of parameter, Goodness of fit.**

I. INTRODUCTION

Assuming as the original distribution, the zero-truncated version of can be expressed as

$$P(x;\theta)=\frac{P_0(x;\theta)}{1-P_0(0;\theta)} \quad ;x=1,2,3,... \qquad (1.1)$$

In probability theory, zero-truncated distributions are certain discrete distributions having support the set of positive integers. Zero-truncated distributions are suitable models for modeling data when the data to be modeled originate from a mechanism which generates data excluding zero counts.

Shukla and Shanker (2019) introduced Poisson-Ishita distribution (PID) having probability mass function (pmf)

$$P_0(x;\theta)=\frac{\theta^3}{(\theta^3+2)}\frac{x^2+3x+(\theta^3+2\theta^2+\theta+2)}{(\theta+1)^{x+3}}; \qquad (1.2)$$

$$x=0,1,2,...,\theta>0$$

Statistical properties and mathematical properties of PID for modeling data from biological science can be seen in Shukla and Shanker (2019), whereas Ishita distribution introduced by Shanker and Shukla (2017) with probability density function (pdf)

$$f(x;\theta)=\frac{\theta^3}{\theta^3+2}(\theta+x^2)e^{-\theta x} \quad ;x>0,\ \theta>0 \qquad (1.3)$$

The statistical properties, estimation of parameter and applications of Ishita distribution for modeling lifetime data have been mentioned in Shanker and Shukla (2017). It has also been established that pmf (1.2) gives better fit over Poisson-Akash distribution (PAD) proposed by Shanker (2017a) whereas Ishita distribution has been observed to be better over exponential distribution, Lindley distribution of Lindley (1958) and Akash distribution of Shanker (2015) for modeling lifetime data.

The motivation of the paper arises from the fact that Ishita distribution gives much closer fit over exponential, Lindley, Akash and Sujatha distributions as well as PID gives better fit over Poisson distribution, Poisson-Lindley distribution (PLD),Poisson-Akash distribution (PAD), and Poisson-Sujatha distribution (PSD) it is expected that a zero-truncated Poisson-Ishita distribution (ZTPID) would provide better fit over zero-truncated Poisson distribution(ZTPD), zero-truncated Poisson-Lindley distribution (ZTPLD),zero-truncated Poisson-Akash distribution (ZTPAD) and zero-truncated Poisson-Sujatha distribution (ZTPSD).

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This paper is presented in five sections which are as follows: In the second section, a zero-truncated Poisson-Ishita distribution (ZTPID) has been proposed. In the third section, moments based measures including expressions for coefficient of variation, skewness, kurtosis, and index of dispersion have been derived. Estimation of its parameter using both the method of moments and maximum likelihood estimation has been discussed in the fourth section. In the last section, applications of ZTPID have been discussed on two observed real datasets and its goodness of fit has been established over ZTPD, ZTPLD, ZTPAD and ZTPSD.

II. ZERO-TRUNCATED POISSON-ISHITA DISTRIBUTION

Using (1.1) and (1.2), the pmf of zero-truncated Poisson-Ishita distribution (ZTPID) can be obtained as

$$P(x;\theta)=\frac{\theta^3}{\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2}\frac{x^2+3x+\left(\theta^3+2\theta^2+\theta+2\right)}{(\theta+1)^x}\ ;x=1,2,3,...,\ \theta>0 \qquad (2.1)$$

The ZTPID can also be obtained by compounding size-biased Poisson distribution (SBPD) having pmf

$$g(x\,|\,\lambda)=\frac{e^{-\lambda}\lambda^{x-1}}{(x-1)!}\ ;x=1,2,3,\ldots,\lambda>0 \qquad (2.2)$$

when the parameter λ of SBPD follows a continuous distribution having pdf

$$h(\lambda;\theta)=\frac{\theta^3}{\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2}\left[(\theta+1)^2\lambda^2+2(\theta+1)\lambda+\left(\theta^3+2\theta^2+\theta+2\right)\right]e^{-\theta\lambda}$$

$$,\lambda>0,\theta>0 \qquad (2.3)$$

The pmf of ZTPID is thus obtained as

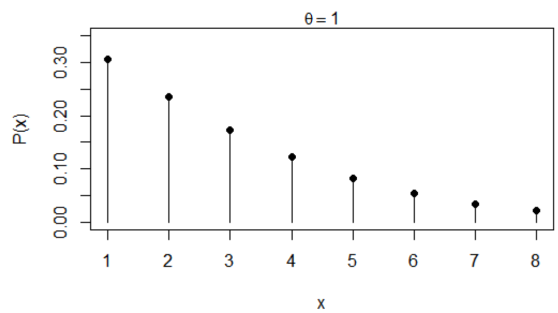
$$P(x;\theta)=\int_0^\infty g(x\,|\,\lambda)\cdot h(\lambda;\theta)\,d\lambda$$

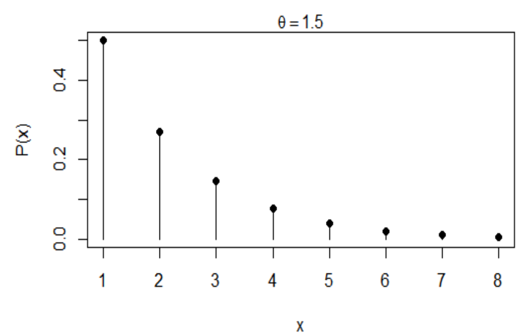
$$=\int_0^\infty\frac{e^{-\lambda}\lambda^{x-1}}{(x-1)!}\cdot\frac{\theta^3}{\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2}\begin{bmatrix}(\theta+1)^2\lambda^2+2(\theta+1)\lambda+\\ \left(\theta^3+2\theta^2+\theta+2\right)\end{bmatrix}e^{-\theta\lambda}d\lambda \qquad (2.4)$$

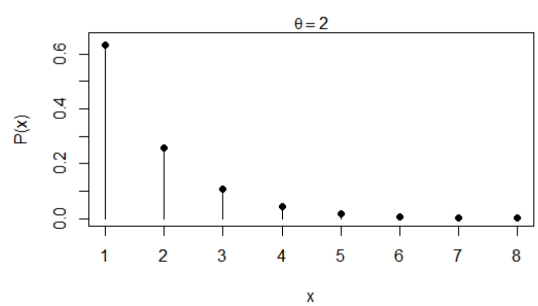
$$=\frac{\theta^3}{\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2}\left[\frac{(x+1)x}{(\theta+1)^x}+\frac{2x}{(\theta+1)^x}+\frac{\theta^3+2\theta^2+\theta+2}{(\theta+1)^x}\right]$$

$$=\frac{\theta^3}{\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2}\frac{x^2+3x+\left(\theta^3+2\theta^2+\theta+2\right)}{(\theta+1)^x};\quad x=1,2,3,...,\ \theta>0$$

which is the pmf of ZTPID with parameter θ. The behavior of the pmf of ZTID has been shown in figure 1.







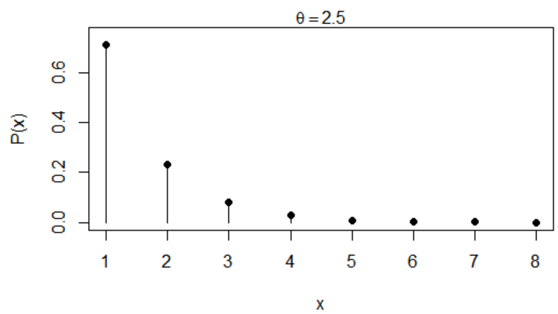


Fig. 1. Behavior of the pmf of ZTPID for varying values of parameter theta

Further it can be easily established that ZTPID is unimodal and has increasing hazard rate. Since

$$\frac{P(x+1;\theta)}{P(x;\theta)}=\left(\frac{1}{\theta+1}\right)\left[1+\frac{2x+4}{x^2+3x+\left(\theta^3+2\theta^2+\theta+2\right)}\right]$$

is a decreasing function of x, $P(x;\theta)$ is log-concave. Therefore, ZTPID is unimodal, has increasing failure rate (IFR), and hence increasing failure rate average (IFRA). It is new better than used (NBU), new better than used in expectation (NBUE), and has decreasing mean residual life (DMRL). Detailed discussions and interrelationships between these aging concepts are available in Barlow and Proschan (1981).

The zero-truncated Poisson- Lindley distribution (ZTPLD), proposed by Ghitany, Al-Mutairi, & Nadarajah (2008) is defined by its pmf

$$P_2(x;\theta)=\frac{\theta^2}{\theta^2+3\theta+1}\frac{x+\theta+2}{(\theta+1)^x} \qquad (2.5)$$

$$;x=1,2,3,....,\theta>0$$

Whereas the Poisson-Lindley distribution (PLD) has been introduced by Sankaran (1970) by compounding Poisson distribution with Lindley distribution proposed by Lindley (1958) Recently, ZTPAD introduced by Shanker (2017b) and its pdf is defined by

$$P_3(x;\theta)=\frac{\theta^3}{\theta^4+2\theta^3+7\theta^2+6\theta+2}\frac{x^2+3x+\left(\theta^2+2\theta+3\right)}{(\theta+1)^x}\;;x=1,2,3,...,\ \theta>0 \qquad (2.6)$$

Various statistical properties, estimation of parameter and applications of ZTPAD are available in Shanker (2017b).

In another study, ZTPSD proposed by Shanker and Hagos (2015), and its pmf is given by

$$P_4(x;\theta)=\frac{\theta^3}{\theta^4+4\theta^3+10\theta^2+7\theta+2}\frac{x^2+(\theta+4)x+\left(\theta^2+3\theta+4\right)}{(\theta+1)^x}\;;x=1,2,3,...,\ \theta>0 \qquad (2.7)$$

The statistical properties, estimation of parameter and applications of ZTPSD are available in Shanker and Hagos (2015).

III. MOMENTS AND MOMENTS BASED MEASURES

The r th factorial moment about origin of ZTPID (2.1) can be obtained as

$$\mu_{(r)}'=E\left[E\left(X^{(r)}\mid\lambda\right)\right]\ ;\text{where } X^{(r)}=X(X-1)(X-2)....(X-r+1)$$

Using (2.4), we have

$$\mu_{(r)}'=\frac{\theta^3}{\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2}\int_0^\infty\left[\sum_{x=1}^{\infty}x^{(r)}\frac{e^{-\lambda}\lambda^{x-1}}{(x-1)!}\right]\cdot\begin{bmatrix}(\theta+1)^2\lambda^2+2(\theta+1)\lambda\\+\left(\theta^3+2\theta^2+\theta+2\right)\end{bmatrix}e^{-\theta\lambda}d\lambda$$

$$=\frac{\theta^3}{\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2}\int_0^\infty\left[\lambda^{r-1}\sum_{x=r}^{\infty}x\frac{e^{-\lambda}\lambda^{x-r}}{(x-r)!}\right]\cdot\begin{bmatrix}(\theta+1)^2\lambda^2+2(\theta+1)\lambda\\+\left(\theta^3+2\theta^2+\theta+2\right)\end{bmatrix}e^{-\theta\lambda}d\lambda$$

Taking $x=x+r$, we get

$$\mu_{(r)}'=\frac{\theta^3}{\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2}\int_0^\infty\left[\lambda^{r-1}\sum_{x=0}^{\infty}(x+r)\frac{e^{-\lambda}\lambda^{x}}{x!}\right]\cdot\begin{bmatrix}(\theta+1)^2\lambda^2+2(\theta+1)\lambda\\+\left(\theta^3+2\theta^2+\theta+2\right)\end{bmatrix}e^{-\theta\lambda}d\lambda$$

$$=\frac{\theta^3}{\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2}\int_0^\infty\lambda^{r-1}(\lambda+r)\cdot\left[(\theta+1)^2\lambda^2+2(\theta+1)\lambda+\left(\theta^3+2\theta^2+\theta+2\right)\right]e^{-\theta\lambda}d\lambda$$

After algebraic simplification, the expression for the r th factorial moment about origin of ZTPID can be expressed as

$$\mu_{(r)}' = \frac{r!(\theta+1)\left[\begin{array}{l} r^2\left(\theta^2+2\theta+1\right)+r\left(3\theta^2+6\theta+3\right)+ \\ \left(\theta^5+2\theta^4+\theta^3+2\theta^2+4\theta+2\right) \end{array}\right]}{\theta^r\left(\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2\right)}; \qquad (3.1)$$

$r = 1, 2, 3, \ldots$

Substituting $r = 1, 2, 3, \text{and } 4$ in equation (3.1), the first four factorial moments about origin can be obtained and using the relationship between moments about origin and factorial moments about origin, the first four moments about origin of ZTPID can be obtained as

$$\mu_1' = \frac{\theta^6+3\theta^5+3\theta^4+7\theta^3+18\theta^2+18\theta+6}{\theta\left(\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2\right)}$$

$$\mu_2' = \frac{(\theta+1)(\theta^6+4\theta^5+5\theta^4+8\theta^3+36\theta^2+54\theta+24)}{\theta^2\left(\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2\right)}$$

$$\mu_3' = \frac{(\theta+1)(\theta^7+8\theta^6+19\theta^5+24\theta^4+90\theta^3+270\theta^2+312\theta+120)}{\theta^3\left(\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2\right)}$$

$$\mu_4' = \frac{\begin{array}{l}(\theta+1)(\theta^8+16\theta^7+65\theta^6+116\theta^5+264\theta^4+ \\ 1086\theta^3+2328\theta^2+2160\theta+720)\end{array}}{\theta^4\left(\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2\right)}$$

Again using the relationship between moments about origin and moments about mean, the moments about mean of ZTPID are thus obtained as

$$\mu_2 = \sigma^2 = \frac{(\theta+1)\left(\begin{array}{l}\theta^{10}+4\theta^9+6\theta^8+27\theta^7+69\theta^6+98\theta^5 \\ +136\theta^4+208\theta^3+180\theta^2+72\theta+12\end{array}\right)}{\theta^2\left(\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2\right)^2}$$

$$\mu_3 = \frac{(\theta+1)\left(\begin{array}{l}\theta^{16}+8\theta^{15}+27\theta^{14}+74\theta^{13}+257\theta^{12}+742\theta^{11}+1537\theta^{10} \\ +2898\theta^9+5110\theta^8+7054\theta^7+8144\theta^6+8848\theta^5+ \\ 7884\theta^4+4872\theta^3+1944\theta^2+456\theta+48\end{array}\right)}{\theta^3\left(\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2\right)^3}$$

$$\mu_4 = \frac{\left(\begin{array}{l}\theta^{22}+17\theta^{21}+109\theta^{20}+410\theta^{19}+1398\theta^{18}+5032\theta^{17}+15262\theta^{16} \\ +37746\theta^{15}+85637\theta^{14}+178055\theta^{13}+319845\theta^{12}+514060\theta^{11} \\ +765908\theta^{10}+1015912\theta^9+1178008\theta^8+1227824\theta^7+ \\ 1133224\theta^6+858928\theta^5+494640\theta^4 \\ +203616\theta^3+56208\theta^2+9360\theta+720\end{array}\right)}{\theta^4\left(\theta^5+2\theta^4+\theta^3+6\theta^2+6\theta+2\right)^4}$$

Finally, the coefficient of variation (C.V), coefficient of Skewness $\left(\sqrt{\beta_1}\right)$, and coefficient of Kurtosis $\left(\beta_2\right)$ of ZTPID are obtained as

$$C.V. = \frac{\sigma}{\mu_1'} = \frac{\sqrt{\begin{array}{l}(\theta+1)(\theta^8+2\theta^7+\theta^6+18\theta^5+ \\ 32\theta^4+16\theta^3+72\theta^2+48\theta+12)\end{array}}}{(\theta^5+2\theta^4+\theta^3+6\theta^2+12\theta+6)}$$

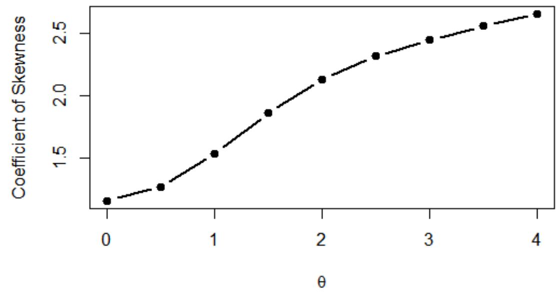
$$\sqrt{\beta_1} = \frac{\mu_3}{\left(\mu_2\right)^{3/2}} = \frac{\left(\begin{array}{l}\theta^{14}+6\theta^{13}+14\theta^{12}+40\theta^{11}+163\theta^{10}+ \\ 376\theta^9+622\theta^8+1278\theta^7 \\ +1932\theta^6+1912\theta^5+2388\theta^4+ \\ 2160\theta^3+1176\theta^2+360\theta+48\end{array}\right)}{\left((\theta+1)\left(\begin{array}{l}\theta^8+2\theta^7+\theta^6+18\theta^5+32\theta^4+ \\ 16\theta^3+72\theta^2+48\theta+12\end{array}\right)\right)^{3/2}}$$

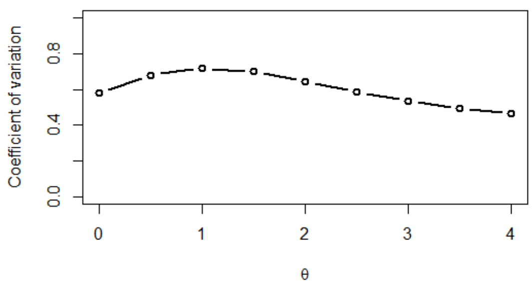
$$\beta_2 = \frac{\mu_4}{\mu_2^{\ 2}} = \frac{\left(\begin{array}{l}\theta^{20}+15\theta^{19}+78\theta^{18}+239\theta^{17}+842\theta^{16}+ \\ 3109\theta^{15}+8202\theta^{14}+18233\theta^{13}+40969\theta^{12} \\ +77884\theta^{11}+123108\theta^{10}+189960\theta^9+ \\ 262880\theta^8+300192\theta^7+314744\theta^6 \\ +298144\theta^5+222192\theta^4+116400\theta^3+ \\ 39648\theta^2+7920\theta+720\end{array}\right)}{(\theta+1)\left(\begin{array}{l}\theta^8+2\theta^7+\theta^6+18\theta^5+32\theta^4+ \\ 16\theta^3+72\theta^2+48\theta+12\end{array}\right)^2}$$

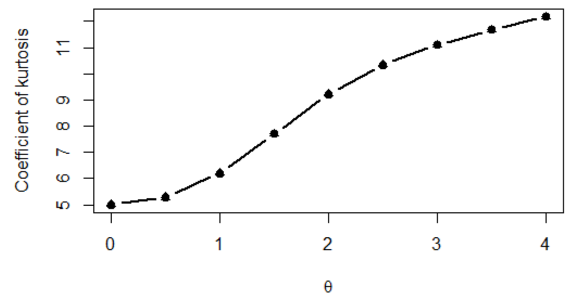
The index of dispersion of ZTPAD is given by

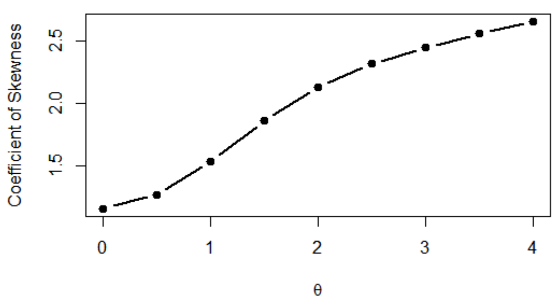
$$\gamma = \frac{\sigma^2}{\mu} = \frac{\begin{array}{l}(\theta^8+2\theta^7+\theta^6+18\theta^5+32\theta^4+ \\ 16\theta^3+72\theta^2+48\theta+12)\end{array}}{\theta\left(\theta^3+6\right)\left(\begin{array}{l}\theta^5+2\theta^4+\theta^3+ \\ 6\theta^2+6\theta+2\end{array}\right)}$$

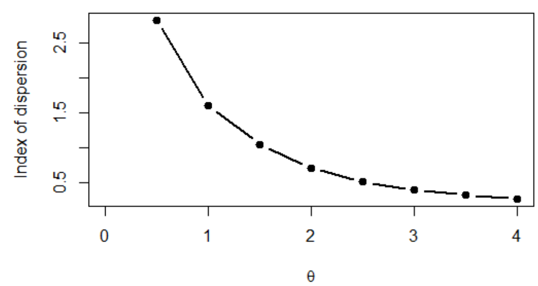
It can be easily verified that the ZTPID is over dispersed $\left(\mu<\sigma^2\right)$, equi-dispersed $\left(\mu=\sigma^2\right)$ and under dispersed $\left(\mu>\sigma^2\right)$ for $\theta<(=)>\theta^*=1.55383$ respectively. Further, ZTPLD is over dispersed $\left(\mu<\sigma^2\right)$, equi-dispersed $\left(\mu=\sigma^2\right)$ and under dispersed $\left(\mu>\sigma^2\right)$ for $\theta<(=)>\theta^*=1.258627$ respectively. The behaviour of coefficient of variation, skewness, kurtosis and index of dispersion (ID) are shown in figure 2











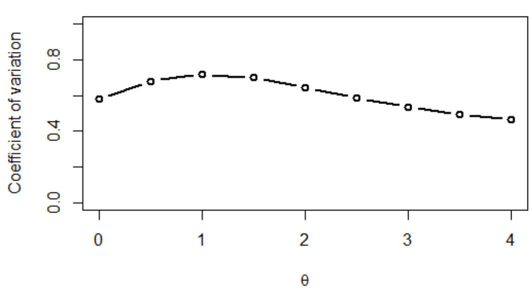


Fig.2. behavior of CV, $\sqrt{\beta_1}$, β_2 and ID of ZTPID.

III. ESTIMATION OF PARAMETER

A. *Method of Moment Estimate (MOME):*

Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from the ZTPID (2.1). Equating the population mean to the corresponding sample mean, the MOME $\tilde{\theta}$ of θ is the solution of the following non-linear equation

$$(\bar{x}-1)\theta^6+3(\bar{x}-1)\theta^5+(\bar{x}-3)\theta^4+(6\bar{x}-7)\theta^3+6(\bar{x}-3)\theta^2+2(\bar{x}-9)\theta-6=0$$

, where $\bar{x}$ is the sample mean.

B. *Maximum Likelihood Estimate (MLE):*

Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from the ZTPID (2.1) and let f_x be the observed frequency in the sample corresponding to $X = x(x = 1, 2, 3, ..., k)$ such that $\sum_{x=1}^{k} f_x = n$, where k is the largest observed value having non-zero frequency. The likelihood function L of the ZTPAD is given by

$$L = \left(\frac{\theta^3}{\theta^5 + 2\theta^4 + \theta^3 + 6\theta^2 + 6\theta + 2} \right)^n \frac{1}{(\theta+1)^{\sum_{x=1}^{k} x f_x}} \prod_{x=1}^{k} \left[x^2 + 3x + \left(\theta^3 + 2\theta^2 + \theta + 2 \right) \right]^{f_x}$$

The log likelihood function is given by

$$\log L = n \log\left(\frac{\theta^3}{\theta^5 + 2\theta^4 + \theta^3 + 6\theta^2 + 6\theta + 2} \right) - \sum_{x=1}^{k} x\, f_x \log(\theta+1) + \sum_{x=1}^{k} f_x \log\left[x^2 + 3x + \left(\theta^3 + 2\theta^2 + \theta + 2 \right) \right]$$

and the log likelihood equation is thus obtained as

$$\frac{d \log L}{d\theta} = \frac{3n}{\theta} - \frac{n\left(5\theta^4 + 8\theta^3 + 3\theta^2 + 12\theta + 6\right)}{\theta^5 + 2\theta^4 + \theta^3 + 6\theta^2 + 6\theta + 2} - \frac{n\bar{x}}{\theta+1} + \sum_{x=1}^{k} \frac{(3\theta^2 + 4\theta + 1) f_x}{x^2 + 3x + \left(\theta^3 + 2\theta^2 + \theta + 2 \right)}$$

The maximum likelihood estimate $\hat{\theta}$ of θ is the solution of the following non linear equation

$$\frac{3n}{\theta} - \frac{n\left(5\theta^4 + 8\theta^3 + 3\theta^2 + 12\theta + 6\right)}{\theta^5 + 2\theta^4 + \theta^3 + 6\theta^2 + 6\theta + 2} - \frac{n\bar{x}}{\theta+1} + \sum_{x=1}^{k} \frac{(3\theta^2 + 4\theta + 1) f_x}{x^2 + 3x + \left(\theta^3 + 2\theta^2 + \theta + 2 \right)} = 0 \quad ,$$

where $\bar{x}$ is the sample mean.This non-linear equation can be solved by any numerical iteration methods such as Newton-Raphson method, Bisection method, Regula –Falsi method etc. R-software has been used to solve the above non-linear equation to estimate the parameter.

IV. APPLICATIONS

The ZTPID has been applied on two real count datasets, first dataset due to Finney and Varley (1955) for number of counts of flower and second data related to number of European red mites on apple leaves due to Garman (1923) . It is obvious from the values of Chi-square $\left(\chi^2\right)$ and p-values that ZTPID gives much closer fit than ZTPD, ZTPLD, ZTPAD and ZTPSD. Therefore, ZTPID can be considered an important tool for modeling count data excluding zero-count.

Table I: The numbers of counts of flower heads as per the number of fly eggs reported by Finney and Varley (1955)

Number of fly eggs	Observed Frequency	Expected Frequency				
		ZTPD	**ZTPLD**	**ZTPSD**	**ZTPAD**	**ZTPID**
1	22	15.3	26.8	26.3	25.1	**24.9**
2	18	21.9	19.8	19.8	19.8	**19.7**
3	18	20.8	13.9	14.1	14.7	**14.7**
4	11	14.9	9.5	9.7	10.3	**10.3**
5	9	8.5	6.4	6.5	6.9	**6.9**
6	6	4.1	4.2	4.2	4.4	4.5
7	3	1.7	2.7	2.7	2.8	2.8
8	0	0.6	1.7	1.7	1.7	1.7
9	1	0.3	3.0	2.9	2.3	2.6
Total	88	88.0	88.0	88.0	88.0	**88.0**
ML estimate		$\hat{\theta} = 2.86040$	$\hat{\theta} = 0.71855$	$\hat{\theta} = 0.98137$	$\hat{\theta} = 1.021503$	$\hat{\theta} = 1.01407$
χ^2		6.677	3.743	2.76	2.12	**2.10**
d.f.		4	4	4	4	**4**
p-value		**0.1540**	**0.4419**	**0.5987**	**0.7137**	**0.7174**

Table II: Number of European red mites on apple leaves, reported by Garman (1923)

Number of European red mites	Observed Frequency	Expected Frequency				
		ZTPD	ZTPLD	ZTPSD	ZTPAD	ZTPID
1	**38**	**28.7**	**36.2**	**35.5**	**35.8**	**36.2**
2	17	25.7	20.4	20.8	20.5	20.2
3	10	15.3	11.2	11.5	11.4	11.1
4	9	6.9	5.9	6.1	6.1	6.0
5	3	2.5	3.1	3.1	3.1	3.2
6	2	0.7	1.6	1.5	1.6	1.6
7	1	0.2	0.8	0.8	0.8	0.8
8	0	0.1	0.8	0.7	0.7	0.9
Total	80	80.0	80.0	80.0	80.0	80.0
ML Estimate		$\hat{\theta}=1.791615$	$\hat{\theta}=1.185582$	$\hat{\theta}=1.53951$	$\hat{\theta}=1.575472$	$\hat{\theta}=1.49244$
χ^2		9.827	2.427	2.561	2.26	2.24
d.f.		2	3	3	3	3
P-value		0.0073	0.4886	0.4644		0.5202

Profile plots of ZTPID for parameter and fitted probability plots on considered dataset are given in fig.3, 4, 5 and 6

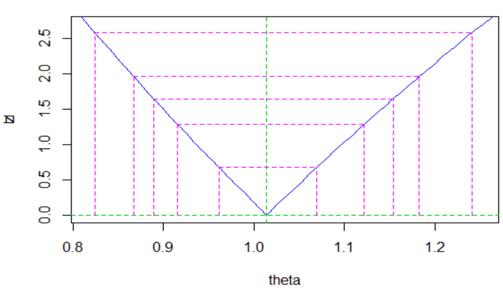


Fig. 3. Profile plot of estimated parameter for the first dataset

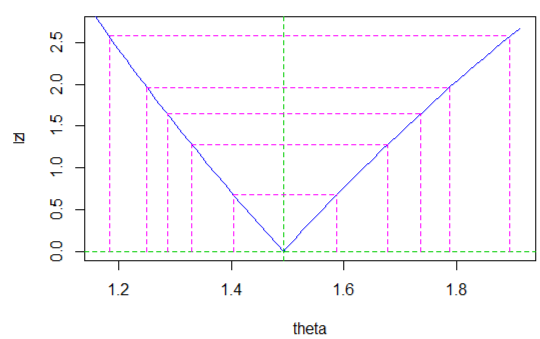


Fig.4. Profile plot of estimated parameter for the second dataset.

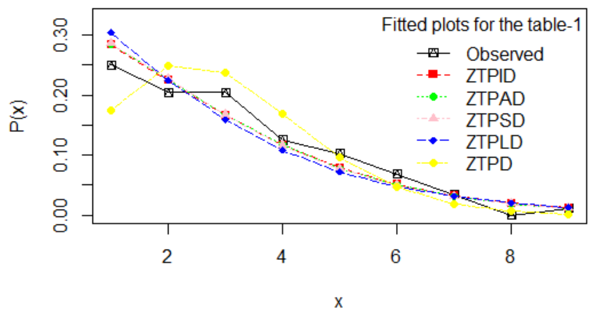


Fig.5. Fitted probability plot for table-1

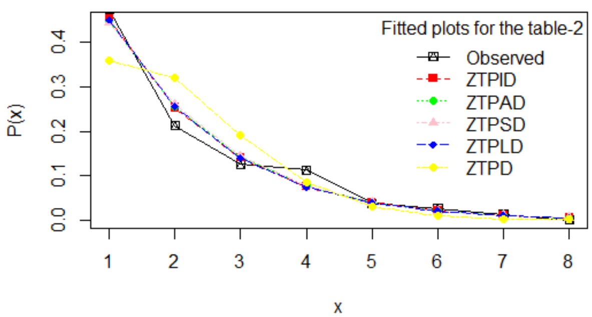


Fig.6. Fitted probability plot for table-2

CONCLUDING REMARKS

A ZTPID has been proposed. Moment based measure including coefficient of variation, skewness, kurtosis, and the index of dispersion have been obtained. The method of maximum likelihood and the method of moments have also been discussed for estimating its parameter. Two examples of count datasets have been illustrated to test its goodness of fit. The ZTPID has been found satisfactory on both datasets over ZTPD, ZTPAD and ZTPSD.

ACKNOWLEDGMENT

Authors are thankful to the Chief editor as well as reviewers for given valuable comments to improve the quality of paper.

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