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Inelastic Cable-connected Satellites System under Several Influences of General Nature: Equations of Motion in Elliptical Orbit

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Abstract: **We derive a set of non-linear, non-homogeneous and non-autonomous differential equations for the motion of a system of two inelastic cable-connected artificial satellites under the influence of shadow of the earth, solar radiation pressure, oblateness of the earth, air resistance and earth's magnetic field. The motion of the system is studied relative to its center of mass which has been assumed to move along a Keplerian elliptical orbit. The equation of relative motion of the system has been obtained. Equations of motion have been obtained in Rotating frame of reference and thereafter in Nechvile's Co-ordinate System. Applying simulations, the equations of motion lead to Jacobian integral of motion of the system. Further simulations and the equations of motion give rise to one equilibrium position of motion of the system concerned under the above mentioned perturbative forces.**

Index Terms: Elliptical orbit, Equations of motion, Inelastic cable, Nechvile's co-ordinate system, Rotating frame of reference, Two satellites system.

I. INTRODUCTION

Beletsky and Novikova (1969) studied the motion of a system of two cable-connected satellites in the central gravitational field of force relative to its center of mass. This study assumed that the two satellites are moving in the plane of the center of mass. Singh and Demin (1972) and Singh (1973) investigated the problem in two and three dimensional cases. Das et al. (1976) studied the effect of magnetic force on the motion of a system of two cable-connected satellites in orbit. Sinha and Singh (1987) studied effect of solar radiation pressure on the motion and stability of the system of two inter connected satellites when their center of mass moves in circular orbit. Again, Sinha and Singh (1988) could generalize the above problem by considering the center of mass of the system moving in elliptical orbit. Beletsky and Levin (1993) studied dynamics of space tether's systems. Kurpa et al. (2000) explained about a new concept of space flight for tethered satellite systems. Again Kurpa et al. (2006) studied about modelling, dynamics and control of tethered satellite systems. Sharma and Narayan (2001) studied non – linear oscillation of inter connected satellite system under the combined influence of the solar radiation pressure and dissipative forces of general nature. Again Sharma and Narayan (2002) investigated effect of solar radiation pressure on the motion and stability of inter connected satellites system in orbit. Singh et. al. (2001) studied non- linear effects on the motion and stability of an inter connected satellites system orbiting around an oblate Earth. Celled and Sidorenko (2008) studied some properties of dumb-bell satellite altitude dynamics. Umar (2013) explained development of satellite technology and its impact on social life. Kumar and Srivastava (2006) studied evolutional and non-evolutional motion of a system of two cable-connected artificial satellites under some perturbative forces. Kumar and Prasad (2015) studied about non-linear planer oscillation of a cable–connected satellites system and non–resonance. Kumar and Kumar (2016) studied equilibrium positions of a cable-connected satellites system under several influences. Kumar (2018) studied liberation points of a cable–connected satellites system under the influence of solar radiation pressure, earth's magnetic field, shadow of the earth and air resistance: circular orbit.

The work is a physical and mathematical idealization of real space system. We establish the equations of motion of the system under the influence of shadow of the earth, solar

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radiation pressure, oblateness of the earth, air resistance and earth's magnetic field. The influence of the above mentioned perturbations on the system has been studied singly and by a combination of any two or three or four of them by various workers, but never conjointly all at a time. Therefore, these could not give a real picture of motion of the system. This fact has initiated the present research work. The satellites are connected by a light, flexible, inextensible and non-conducting string. Central attractive force of the earth will be the main force and all other forces, being small enough are considered here as perturbing forces. Since masses of the satellites are small and distances between the satellites and other celestial bodies are very large, the gravitational forces of attraction between the satellites and other celestial bodies including the sun have been neglected. The satellites are considered as charged material particles. Here, nutation and wobbling of the orbit of the center of mass of the system are not taken into account.

II. EQUATION OF MOTION OF THE CENTRE OF MASS OF THE SYSTEM

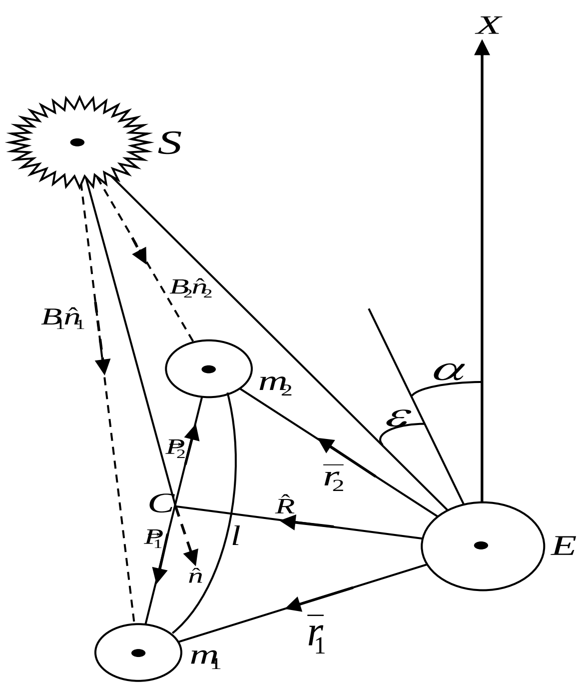

Fig. 1: Diagrammatical representation of the cable-connected satellites system under several influences

From the first principle of physics, we write the Lagrange's equations of motion of the first kind for the system as

$$m_1\ddot{\bar{r}}_1 = -\mu\frac{m_1\bar{r}_1}{r_1^3} + \lambda(\bar{r}_1 - \bar{r}_2) + Q_1(\dot{\bar{r}}_1 \times \bar{H}) + \gamma B_1\bar{n}_1 - 3\frac{m_1\mu k_2}{r_1^5}\bar{r}_1 - \rho_a c_1 m_1|\dot{\bar{r}}_1|\dot{\bar{r}}_1 \qquad (1)$$

and

$$m_2\ddot{\bar{r}}_2 = -\mu\frac{m_2\bar{r}_2}{r_2^3} + \lambda(\bar{r}_2 - \bar{r}_1) + Q_2(\dot{\bar{r}}_2 \times \bar{H}) + \gamma B_2\bar{n}_2 - 3\frac{m_2\mu k_2}{r_2^5}\bar{r}_2 - \rho_a c_2 m_2|\dot{\bar{r}}_2|\dot{\bar{r}}_2 \qquad (2)$$

with the condition of constraint

$$|\bar{r}_1 - \bar{r}_2| \leq \ell \qquad (3)$$

If the inequality sign holds good in (3), then the system moves without any constraint. This is called "Free motion" of the system. If equality sign holds good, then the system moves with the active constraint. This is called "Constrained motion". But in practice, motion of the system is a combination of free and constrained motion. It is based on the fact that with the lapse of time constrained motion of the system gets converted into free motion.

Where m_1 and m_2 are masses of the two satellites. $\bar{r}_1$ and $\bar{r}_2$ are the radius vectors of the particles m_1 and m_2 respectively with respect to the earth's center E as shown in fig. 1. μ is the product of mass of the earth and gravitational constant. λ is undetermined Lagrange's multiplier arising due to constraint for the finite length of the cable. Q_1, Q_2 are charges of the two satellites. $\bar{H}$ is the intensity of earth's magnetic field. γ is a shadow function which depends on the illumination of the system of satellites by the sun rays. If γ is equal to zero, then the system is affected by the shadow of the earth. If γ is equal to one, then the system is not within the said shadow. B_1 and B_2 are the absolute values of the forces due to the direct solar pressure on m_1 and m_2 respectively. $\bar{n}_1$ and $\bar{n}_2$ are the unit vectors in the direction of the sun rays towards m_1 and m_2 respectively. c_1 and c_2 are the Ballistic coefficients. ρ_a is the average density of the atmosphere. ℓ is the length of the cable connecting m_1 and m_2.

Also,

$$k_2 = \frac{\varepsilon R_e^2}{3},\ \varepsilon = \alpha_R - \frac{m}{2},\ m = \frac{\Omega^2 R_e}{g_e} \qquad (4)$$

α_R is the earth's oblateness. Ω is angular velocity of the earth's rotation. R_e is equatorial radius of the earth. g_e is the force of gravity.

As $\bar{n}_1$ and $\bar{n}_2$ are almost parallel, we replace them by $\bar{n}$. Adding the equations (1) and (2), we obtain

$$m\ddot{\bar{R}} = -\mu\left[\frac{m_1(\bar{R}+\bar{\rho}_1)}{(R+\rho_1)^3} + \frac{m_2(\bar{R}+\bar{\rho}_2)}{(R+\rho_2)^3}\right] + (Q_1\dot{\bar{r}}_1 + Q_2\dot{\bar{r}}_2)\times\bar{H} + \gamma(B_1+B_2)\bar{n}$$

$$-3\mu k_2\left[\frac{m_1(R+\bar\rho_1)}{(R+\rho_1)^5}+\frac{m_2(R+\bar\rho_2)}{(R+\rho_2)^5}\right]$$
$$-\rho_a\left[c_1m_1\left|(\dot R+\dot{\bar\rho}_1)\right|(\dot R+\dot{\bar\rho}_1)+c_2m_2\left|(\dot R+\dot{\bar\rho}_2)\right|(\dot R+\dot{\bar\rho}_2)\right] \quad (5)$$

Where, $m=(m_1+m_2)$, total mass of the system

$R=\left(\frac{m_1r_1+m_2r_2}{m_1+m_2}\right)$ radius vector of center of mass of the system, with respect to the origin of the attracting center.

$r_j=R+\bar\rho_j$ (j = 1, 2)

$\bar\rho_j=$ radius vector of the particle m_j with respect to the center of mass of the system C as shown in fig. 1.

(6)

We are interested in relative motion of the system with respect to the center of mass. The center of mass is the origin of the frame of reference. Hence, we shall have

$$m_1\bar\rho_1+m_2\bar\rho_2=0 \quad (7)$$

Expanding equation (5) in power of $\frac{\rho_j}{R}$ upto the first order of infinitesimals, we obtain

$$m\ddot R+\frac{\mu mR}{R^3}=F_1+F_2+(Q_1+Q_2)(\dot R\times H)+\gamma(B_1+B_2)n$$
$$-3\mu k_2\left[\frac{m_1(R+\bar\rho_1)}{(R+\rho_1)^5}+\frac{m_2(R+\bar\rho_2)}{(R+\rho_2)^5}\right]-a_1\dot R$$

(8)

Where,

$$F_1=\frac{3\mu}{R^3}(m_1\bar\rho_1+m_2\bar\rho_2)+\frac{3\mu}{R^5}\left[R(m_1\bar\rho_1+m_2\bar\rho_2)\right].R$$
$$F_2=\frac{3\mu}{2R^3}\left[m_1\left\{\left(\frac{\rho_1}{R}\right)^2-5\left(\frac{R}{R}.\frac{\bar\rho_1}{R}\right)^2\right\}+m_2\left\{\left(\frac{\rho_2}{R}\right)^2-5\left(\frac{R}{R}.\frac{\bar\rho_2}{R}\right)^2\right\}.R\right]$$
$$+\frac{3\mu m_1}{R^5}(R.\bar\rho_1)\bar\rho_1+\frac{3\mu m_2}{R^5}(R.\bar\rho_2)\bar\rho_2$$

and

$$a_1=\rho_aR(c_2-c_1)\left(\frac{m_1}{m_1+m_2}\right) \quad (9)$$

Due to the above relation (7), we write

$$F_1=0 \quad (10)$$

Since $\rho_j\ll r_j$, $r_j\sim R$ and $\frac{\rho_j}{R}\ll 1$, we write

$$F_2=0 \quad (11)$$

On the other hand, k_2 is very small. Now using (10) and (11) and neglecting the terms of k_2, we write equation (8) as

$$m\ddot R+\frac{\mu mR}{R^3}+(Q_1+Q_2)(\dot R\times H)-\gamma(B_1+B_2)n+a_1\dot R=0 \quad (12)$$

Again, the effect of air resistance on the motion of the system may be neglected as it is of perturbative nature. Therefore, we write (12) as

$$m\ddot R+\frac{\mu mR}{R^3}+(Q_1+Q_2)(\dot R\times H)-\gamma(B_1+B_2)n=0 \quad (13)$$

Taking dot product of both the sides of equation (13) with $\dot R$, we obtain

$$m\ddot R.\dot R+\frac{\mu mR.\dot R}{R^3}=0 \quad (14)$$

Where

$$n.\dot R=0 \quad (15)$$

(15) is the usual case for earth's satellites.

Integrating (14) we get,

$$\dot R^2=A+\frac{2\mu}{R} \quad (16)$$

A = constant of Integration.

It shows that the center of mass of the system describes an elliptical orbit. In fact, (16) is the equation of energy.

III. EQUATION OF RELATIVE MOTION OF THE SYSTEM

In order to obtain relative equation of motion of the system with respect to the center of mass, we subtract equation (2) from equation (1) and get

$$\left[\ddot r_1-\ddot r_2\right]=-\mu\left[\frac{r_1}{r_1^3}-\frac{r_2}{r_2^3}\right]+(r_1-r_2)\left[\frac{\lambda}{m_1}+\frac{\lambda}{m_2}\right]$$
$$+\left[\frac{Q_1}{m_1}(\dot r_1\times H)-\frac{Q_2}{m_2}(\dot r_2\times H)\right]+\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)n$$
$$-3\mu k_2\left[\frac{r_1}{r_1^5}-\frac{r_2}{r_2^5}\right]-\rho_a\left[c_1|\dot r_1|\dot r_1-c_2|\dot r_2|\dot r_2\right] \quad (17)$$

From (6) and (7), we have

$$\left[\ddot r_1-\ddot r_2\right]=-\left(\frac{m_1+m_2}{m_1}\right)\ddot{\bar\rho}_2 \quad (18)$$

Therefore, vector equation of motion of m_2 can be written as

$$\ddot{\bar\rho}_2+\frac{\mu}{R^3}\bar\rho_2-\frac{3\mu R}{R^5}(R.\bar\rho_2)-\left(\frac{m_1+m_2}{m_1m_2}\right)\lambda\bar\rho_2$$

$$=-\left(\frac{m_1}{m_1+m_2}\right)\left(\frac{Q_1}{m_1}-\frac{Q_2}{m_2}\right)\left[\frac{\mu_E}{R^3}\dot{R}\times\bar{k}_E-3(\bar{k}_E.\bar{e}_r)\dot{R}\times\bar{e}_r\right]+$$
$$\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)n+\frac{3\mu k_2}{R^5}.\bar{\rho}_2-\frac{15\mu k_2}{R^3}\left[R\left(R.\bar{\rho}_2\right)\right]-a_1\dot{R} \tag{19}$$

Where,

$$H=\left[\frac{\mu_E}{R^3}K_E-3\left(K_E.e_r\right)e_r\right] \tag{19 a}$$

K_E is the unit vector along the axis of magnetic dipole of the earth.

e_r is a unit vector along the radius vector $\bar{R}$.

μ_E is the value of the magnetic moment of the earth's dipole.

(19) describes the relative motion of the satellite of mass m_2. The motion of the satellite of mass m_1 can be easily determined with the help of (7).

Our ultimate aim is to obtain the equations of motion of the system in Cartesian form. Therefore, we shall make use of Cartesian transformations and substitutions in the next part of the work.

Let (X,Y,Z) be the co-ordinates of the satellite m_2 with the origin at the center of mass. Next, we write the most common expressions as follows

$$K_E=(\sin\omega\cos v)\bar{i}+(\cos\omega\cos v)\bar{j}+(\cos i)\bar{k}$$
$$B_1n=-B_1\cos\in\cos\alpha\bar{i}-B_1\cos\in\sin\alpha\bar{j}+B_1\sin\in\bar{k}$$
$$B_2n=-B_2\cos\in\cos\alpha\bar{i}-B_2\cos\in\sin\alpha\bar{j}+B_2\sin\in\bar{k} \tag{20}$$

Where ω is argument of the perigee. v is true anomaly of the center of mass of the system. i is inclination of the orbit with the equatorial plane. $\in$ is inclination of the oscillatory plane of the masses m_1 and m_2 with the orbital plane of the center of mass of the system. α is inclination of the ray. Now, we can easily write the Cartesian equivalent of the vector equation (19) as

$$\ddot{X}+\frac{\mu}{R^3}X-\frac{3\mu}{R^3}\cos v\left(X\cos v+Y\sin v\right)-\left(\frac{m_1+m_2}{m_1m_2}\right)\lambda X=$$
$$-\left(\frac{m_1}{m_1+m_2}\right)\left(\frac{Q_1}{m_1}-\frac{Q_2}{m_2}\right)\left[\frac{\mu_E}{R^3}\cos i(\dot{R}\sin v+\dot{v}R\cos v)\right]+$$
$$\gamma\left(\frac{B_2}{m_2}-\frac{B_1}{m_1}\right)\cos\in\cos\alpha-\frac{3\mu k_2}{R^5}X+$$
$$\frac{15\mu k_2}{R^5}\cos v\left(X\cos v+Y\sin v\right)-a_1\left(\dot{R}\cos v-R\dot{v}\sin v\right)$$

$$\ddot{Y}+\frac{\mu}{R^3}Y-\frac{3\mu}{R^3}\sin v\left(X\cos v+Y\sin v\right)-\left(\frac{m_1+m_2}{m_1m_2}\right)\lambda Y=$$
$$-\left(\frac{m_1}{m_1+m_2}\right)\left(\frac{Q_1}{m_1}-\frac{Q_2}{m_2}\right)\left[\frac{\mu_E}{R^3}\cos i\left(\dot{v}R\sin v+\dot{R}\cos v\right)\right]+$$
$$\gamma\left(\frac{B_2}{m_2}-\frac{B_1}{m_1}\right)\cos\in\sin\alpha-\frac{3\mu k_2}{R^5}Y+$$
$$\frac{15\mu k_2}{R^5}\sin v\left(X\cos v+Y\sin v\right)-a_1\left(\dot{R}\sin v+R\dot{v}\cos v\right)$$

and

$$\ddot{Z}+\frac{\mu}{R^3}Z-\left(\frac{m_1+m_2}{m_1m_2}\right)\lambda Z=-\left(\frac{m_1}{m_1+m_2}\right)\left(\frac{Q_1}{m_1}-\frac{Q_2}{m_2}\right)$$
$$\left[\frac{\mu_E}{R^3}\left\{\dot{R}\cos(v+\omega)-R\dot{v}\sin(v+\omega)\right\}+3\sin(v+\omega)R\dot{v}\right]\sin i$$
$$+\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\sin\in-\frac{3\mu k_2}{R^5}Z \tag{21}$$

The condition of constraint (3) becomes

$$X^2+Y^2+Z^2\leq 1 \tag{22}$$

IV. EQUATIONS OF MOTION IN ROTATING FRAME OF REFERENCE

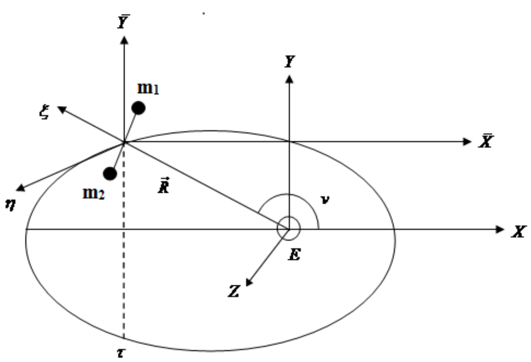

Fig. 2: The cable-connected satellites system and the rotating frame of reference

In space, all the celestial bodies are in motion. Therefore, we consider the rotating frame of reference to understand the exact picture of motion of the system concerned. We transform equations of motion (21) from old frame of reference to new rotating frame of reference. For this, we put the general substitutions for the required transformation as

$$X=\left(\xi\cos v-\eta\sin v\right),$$
$$Y=\left(\xi\sin v+\eta\cos v\right)$$

and

$$Z=\tau \qquad (23)$$

where, axis ξ is along the radius vector R, axis $\eta$is towards the transversal to the orbit of the center of mass in the direction of motion and axis $\tau$is being directed along the normal to the orbital plane of the center of mass of the system.

With the help of (23), we write (21) as

$$\xi-2v\eta-v\eta-v^2\xi-\frac{2\mu}{R^3}\xi-\left(\frac{m_1+m_2}{m_1m_2}\right)\lambda\xi=$$
$$-\left(\frac{m_1}{m_1+m_2}\right)\left(\frac{Q_1}{m_1}-\frac{Q_2}{m_2}\right)\left(\frac{\mu_E}{R^3}\right)(\cos i)Rv+$$
$$\gamma\left(\frac{B_2}{m_2}-\frac{B_1}{m_1}\right)\cos\in\cos(v-\alpha)+\frac{12\mu k_2}{R^5}\xi-a_1R$$

$$\eta+2v\xi+v\xi-v^2\eta+\frac{\mu}{R^3}\eta-\left(\frac{m_1+m_2}{m_1m_2}\right)\lambda\eta=$$
$$-\left(\frac{m_1}{m_1+m_2}\right)\left(\frac{Q_1}{m_1}-\frac{Q_2}{m_2}\right)\left(\frac{\mu_E}{R^3}\right)R\cos i+$$
$$\gamma\left(\frac{B_2}{m_2}-\frac{B_1}{m_1}\right)\cos\in\sin(v-\alpha)-\frac{3\mu k_2}{R^5}\eta-a_1Rv$$

and

$$\tau+\frac{\mu}{R^3}\tau-\left(\frac{m_1+m_2}{m_1m_2}\right)\lambda\tau=-\left(\frac{m_1}{m_1+m_2}\right)\left(\frac{Q_1}{m_1}-\frac{Q_2}{m_2}\right)\sin i$$
$$\left[\frac{\mu_E}{R^3}\left\{R\cos(v+\omega)-Rv\sin(v+\omega)\right\}+3\sin(v+\omega)Rv\right]+$$
$$\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\sin\in-\frac{3\mu k_2}{R^5}\tau \qquad (24)$$

The condition of constraint (22) takes the form

$$\xi^2+\eta^2+\tau^2\leq 1 \qquad (25)$$

V. EQUATIONS OF MOTION INNECHVILE'S CO-ORDINATESYSTEM

For further studies of the problem, we introduce Nechvile's co-ordinate system (1926), i.e. a dilation. Interpretation of this transformation is to insert a physical quantity “eccentricity” in the required equations of motion. From the first principle of physics, it is clear that the value of eccentricity determines nature of the orbit. Next, we put

$$\xi=\rho X,\ \eta=\rho Y \text{ and } \tau=\rho Z \qquad (26)$$

Where,

$$\rho=\frac{1}{1+e\cos v} \qquad (27)$$

e is eccentricity of the elliptical orbit of center of mass of the system. $V$is the true anomaly as the independent variable in place of time t. We may deduce a relation for t and v as

$$v=\frac{dv}{dt}=\frac{\sqrt{\mu p}}{p^2}.\frac{1}{\rho^2} \qquad (28)$$

p= focal parameter.

Consequently, $2\rho'v+\rho v=0,\ \rho''v-\rho v+\rho'v'=\frac{\sqrt{\mu p}}{p^2}$

(29)

Putting first and second time derivatives obtained from (26) and the values of (28) and (29) in the system of equations (24) and neglecting the infinitesimals of second and higher order terms, we get

$$X''-2Y'-3X\rho=\lambda_\alpha X-\frac{A}{\rho}\cos i$$
$$-\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\in\cos(v-\alpha)+\frac{12\mu k_2}{R^5}\rho X-f\rho\rho'$$

$$Y''+2X'=\lambda_\alpha Y-\frac{A\rho'}{\rho^2}\cos i$$
$$+\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\in\sin(v-\alpha)-\frac{3\mu k_2}{R^5}\rho Y-f\rho^2$$

and

$$Z''+Z=\lambda_\alpha Z-\left[\frac{A\rho'}{\rho^2}\cos(v+\omega)+\frac{A}{\mu_E}\left(\frac{3p^3\rho^3-\mu_E}{\rho}\right)\sin(v+\omega)\right]\sin i$$
$$+\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\sin\in-\frac{3\mu k_2}{R^5}\rho Z$$

(30)

W here,

$$\lambda_\alpha=\frac{p^3\rho^4}{\mu}\left(\frac{m_1+m_2}{m_1m_2}\right)\lambda,$$
$$A=\left(\frac{m_1}{m_1+m_2}\right)\left(\frac{Q_1}{m_1}-\frac{Q_2}{m_2}\right)\frac{\mu_E}{\sqrt{\mu p}},$$
$$f=\frac{a_1p^3}{\sqrt{\mu p}}$$

(31)

The condition of constraint (25) is now given as

$$X^2+Y^2+Z^2\leq\frac{1}{\rho^2} \quad (32)$$

VI. SIMULATIONS OF THEEQUATIONS OF MOTION OF THE TWO CABLE CONNECTED SATELLITES SYSTEM

Numerical solutions of the set of differential equations (30) of the cable-connected satellites system are not possible. Therefore, we are interested for simulations of equations (30). Here we first of all establish Jacobean integral of motion of the system and there after we study about the equilibrium positions of motion of the system concerned.

Jacobean integral of the system:

For simplicity, we consider only the first two equations of (30) as

$$X''-2Y'-3X\rho=\lambda_\alpha X-\frac{A}{\rho}\cos i$$
$$-\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\in\cos(\nu-\alpha)+\frac{12\mu k_2}{R^5}\rho X-f\rho\rho'$$

$$Y''+2X'=\lambda_\alpha Y-\frac{A\rho'}{\rho^2}\cos i$$
$$+\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\in\sin(\nu-\alpha)-\frac{3\mu k_2}{R^5}\rho Y-f\rho^2 \quad (33)$$

The condition of constraint (32) is now given as

$$X^2+Y^2\leq\frac{1}{\rho^2} \quad (34)$$

We shall discuss the case of circular orbit of center of mass of the system. We put $e=0$, $\rho=1$ and $\rho'=0$ in equations (33) and write

$$X''-2Y'-3X=\lambda_\alpha X-A\cos i-\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right).\cos\in\cos(\nu-\alpha)+\frac{12\mu K_2}{R^5}X$$

and

$$Y''+2X'=\lambda_\alpha Y+\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right).\cos\in\sin(\nu-\alpha)-\frac{3\mu K_2}{R^5}Y-f$$

(35)

with the condition of constraint

$$X^2+Y^2\leq 1 \quad (36)$$

For circular orbit,

$$X^2+Y^2=1;\ \text{whence}\ XX'+YY'=0 \quad (37)$$

The system of two satellites is allowed to pass through the shadow beam during its motion. Let us assume that θ_2 is the angle between the axis of the cylindrical shadow beam and the line joining the center of the earth and the end point of the orbit of the center of mass within the earth's shadow, considering the positive direction towards the motion of the system. The system starts to be influenced by the solar pressure when it makes an angle θ_2 with the axis of the shadow beam and remains under the influence of solar pressure till it makes an angle $(2\pi-\theta_2)$ with the axis of the cylindrical shadow beam. Thereafter, the system will enter the shadow beam and the effect of solar pressure will come to an end.

Next, the small secular and long periodic effects of solar pressure together with the effects of earth's shadow on the system may be analyzed by averaging the periodic terms in (35) with respect to υ from θ_2 to $(2\pi-\theta_2)$ for a period when the system is under the influence of the sun rays directly i.e. $\gamma=1$ and from $-\theta_2$ to $+\theta_2$ for a period when the system passes through the shadow beam i.e. $\gamma=0$.

Thus, after averaging the periodic terms, (35) may be written as

$$X''-2Y'-3X=\beta X-A\cos i+\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right).\frac{\cos\in\cos\alpha\sin\theta_2}{\pi}+\frac{12\mu K_2}{R^5}X$$

and

$$Y''+2X'=\beta Y+\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right).\frac{\cos\in\sin\alpha\sin\theta_2}{\pi}-\frac{3\mu K_2}{R^5}Y-f \quad (38)$$

Where $$\beta=\frac{p^3\lambda}{\mu}\left(\frac{m_1+m_2}{m_1m_2}\right) \quad (39)$$

These equations do not contain the time explicitly, Therefore, Jacobean integral of the motion exists.

Multiplying the first and second equations of (38) by X' and Y' respectively, adding them and then integrating the final equation, we get the Jacobian integral in the form

$$X'^2+Y'^2-3X^2=\beta-2AX\cos i$$
$$+\frac{2}{\pi}\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\in\sin\theta_2\left(X\cos\alpha+Y\sin\alpha\right)$$
$$+\frac{3\mu K_2}{R^5}\left(4X^2-Y^2\right)-2fY+h \qquad (40)$$

The surface of zero velocity can be obtained in the form

$$3X^2-2AX\cos i+\frac{2}{\pi}\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right).$$
$$\cos\in\sin\theta_2\left(X\cos\alpha+Y\sin\alpha\right)$$
$$+\frac{3\mu K_2}{R^5}\left(4X^2-Y^2\right)-2fY+\beta+h=0 \qquad (41)$$

$h=$Constant of Integration called Jacobean constant.

It is, therefore, concluded that satellite m_1 moves inside the boundary of different curves of zero velocity, represented by (41) for different values of Jacobean constant h.

Equilibrium positions of the system:

A set of equations (38) for motion of the system in the rotating frame of reference has been obtained. It is assumed that the system is moving with the effective constraint and the connecting cable of the two satellites always remains tight.

The equilibrium positions of motion of the system are given by the constant values of the co-ordinates in the rotating frame of reference. Let us take

$$X=X_0 \text{ and } Y=Y_0 \qquad (42)$$

X_0 and Y_0 are constant, give the equilibrium positions. Therefore, we get

$$X'=X'_0=0 \quad ; \quad X''=X''_0=0$$
$$Y'=Y'_0=0 \quad ; \quad Y''=Y''_0=0 \qquad (43)$$

Putting (42) and (43) in the set of equations (38), we get

$$(3+\beta)X_0=A\cos i-\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right).\frac{\cos\in\cos\alpha\sin\theta_2}{\pi}-\frac{12\mu K_2}{R^5}X_0$$

and

$$\beta Y_0=f-\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right).\frac{\cos\in\sin\alpha\sin\theta_2}{\pi}+\frac{3\mu K_2}{R^5}Y_0 \qquad (44)$$

Actually, it is very difficult to obtain the solution of (44). Hence, we are compelled to make our approaches with certain limitations. In addition to this, we are interested only in the case of the maximum effect of the earth's shadow on motion of the system.

In the further investigation, we put $\in=0$ and $\alpha=0$ as because $\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)$ or θ_2 cannot be zero. Clearly equations (44) become

$$(3+\beta)X_0=A\cos i-\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right).\frac{\sin\theta_2}{\pi}-\frac{12\mu K_2}{R^5}X_0$$

and

$$\beta Y_0=f+\frac{3\mu K_2}{R^5}Y_0 \qquad (45)$$

All the two equations of (45) are independent of each other.

With the help of the two equations of (45), the equilibrium position is obtained as.

$$[X_0,Y_0]=\left[\frac{A\cos i-\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right).\frac{\sin\theta_2}{\pi}}{\left(3+\beta+\frac{12\mu K_2}{R^5}\right)},\frac{f}{\left(\beta-\frac{3\mu K_2}{R^5}\right)}\right] \qquad (46)$$

VII. RESULT AND DISCUSSION

Aim of the present paper is to obtain equations of motion of a system of two cable connected satellites under the influence of several perturbative forces like shadow of the earth, solar radiation pressure, oblateness of the earth, earth’s magnetic field and air resistance. The cable connecting the two satellites is light, flexible, non - conducting and inelastic in nature. The satellites are considered as charged material particles. As the body of the satellites is made up of metal, the satellites cut magnetic lines of force of the earth. According to Lorentz force charges get developed on the two satellites. But magnitude of the charges is very small. Thus, electrostatic interaction between the satellites is not taken into account. Motion of the system is studied relative to the center of mass. Cable connected satellites system in the space is a physical and mathematical modeling of the real space problems, such as space vehicle and astronaut floating in space, two or multi sectional satellites system connected by a cable, manned space capsule attached to its booster by a cable and spinners to provide artificial gravity for the astronaut and finally two satellites at the same time of rendezvous in order to transport a man successfully to an orbiting station. Many space configurations of inter connected satellites system have been proposed and analyzed like two satellites are connected by a rod i.e. dumbbell satellite, two or more satellites connected by a tether. While investigating the relative motion of the system of two cable connected satellites, it is assumed that the particles are subjected to impacts of absolutely inelastic in nature, when the cable tightened up. Shadow of the earth is supposed to be cylindrical in nature and the system is allowed to pass through the shadow beam. Many authors like Beletsky and Novikova (1969), Beletsky and Levin (1993),Celled and Sidorenko (2008), Kurpa et al. (2000,2006) have discussed many important applications of the system of two cable connected satellites.

Our problem is general in the sense that for the first time, we investigated the influence of several perturbative forces like shadow of the earth, solar radiation pressure, oblateness of the earth, earth's magnetic field and air resistance on the system of two cable-connected satellites. Many classical results of two cable-connected satellites system may be verified from our generalized result (30). For example, works of Sinha and Singh (1987,1988), Khan and Goel (2011), Celled and Sidorenko (2008), Kumar and Srivastava (2006), Kumar and Kumar (2016), Kumar (2018) etc.

CONCLUSION

Equations (30) are a set of non-linear, non-homogeneous and non-autonomous differential equations of motion of a system of two cable-connected artificial satellites under the influence of shadow of the earth, solar radiation pressure, oblateness of the earth, earth's magnetic field and air resistance. These equations have wide applications in solving problems of stability of a cable-connected satellites system in orbit. Applying simulations and using the equations (30), we can easily calculate Jacobean integral of motion of the system concerned either in circular or elliptical orbit. In fact, Jacobean integral is potential energy of the system. With the help of Jacobean integral, we can easily study about the condition regarding constrained motion of the system. We can apply simulations and the set of equations of motion (30) to study Non -resonant oscillation, resonant oscillation and Para metric resonant oscillation of a cable - connected satellites system under the influence of several perturbative forces

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