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An Enhanced Two Phase Sampling Ratio Estimator for Estimating Population Mean

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Abstract: **Through this paper, we suggest an enhanced two-phase sampling ratio type estimator for the efficient estimation of population parameter mean by utilizing known values of moments of an auxiliary variable. The salient features affiliated with the developed estimator characterized by mean squared error and bias is also assessed. In addition, the expression for minimum mean squared error for the optimum values has been obtained. To establish the superiority of suggested estimator, efficiency comparison with some existing estimators has been accomplished. An empirical study to elucidate theoretical results through two real population data sets is also presented as an illustration.**

Index Terms: **Double Sampling, Study Character, Auxiliary Character, Bias, Mean Squared Error**.

I. INTRODUCTION

As we all know, in survey research auxiliary information is the information which is often available in every unit of population from previous experiences, census or administrative database and is strongly associated with the main variable under the reference of study. A diverse range of techniques are present in sampling literature, which focuses on the ways of utilizing auxiliary information for achieving better and efficient results. Since the various types of statistical information serve as a sound basis for decision making in different areas of human activities, it is desirable to construct best and accurate estimation methods.

Various authors have utilized known auxiliary information in variety of ways to devise miscellaneous estimators for precise estimation of different population parameter(s).

In the optimal search of most precise and efficient estimator for population mean several ratio, product, difference, exponential and linear regression estimators have been developed and modified by many survey practitioners. If study and auxiliary variable are closely related and exhibit positive correlation, Cochran (1940 and 1942) suggested ratio estimator. The principal aim of ratio method is to use ratio of two sample means, which are not affected by sampling fluctuations, and thus estimates so obtained are nearest to the true parametric values. On contrary, if study characteristic and auxiliary variable exhibit negative correlation, Robson (1957) and Murthy (1964) proposed product estimators. In general, ratio method has wider applicability because of its simple computational approach of studies. A wide discussion on the ratio method of estimation is duly incorporated in Murthy (1967), Des (1972), Cochran (1977), Sukhatme et al. (1984), Singh and Chaudhary (1997), Mukhopadhyay (2012).

For situations where parametric value of population mean of ancillary variable is already available beforehand, the classical ratio estimator reported in literature for mean estimation is

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}}\bar{X} = \hat{R}\bar{X} \qquad (1.1)$$

In the above Eq. (1.1) $\bar{y}$ and $\bar{x}$ represents the values of sample mean of main and ancillary characteristic. The distinctive properties of classical ratio estimator $\hat{\bar{Y}}_R$ already defined in Eq. (1.1) are as

$$Bias\left(\hat{\bar{Y}}_R\right) = \frac{1-f}{n}\bar{Y}\left(C_X^2 - C_X C_Y \rho\right) \qquad (1.2)$$

$$MSE\left(\hat{\bar{Y}}_R\right) = \frac{1-f}{n}\bar{Y}^2\left(C_Y^2 + C_X^2 - 2C_X C_Y \rho\right) \qquad (1.3)$$

On comparing with usual sample mean estimator, ratio estimator yielded efficient results. Since the long past, many modifications on the classical ratio estimator proposed by Cochran (1940) has been made by carefully utilizing information

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on auxiliary variable in various qualitative and quantitative forms. To see the contribution of some renowned authors, one is referred to see the remarkable works of Sisodia and Dwivedi (1981), Prasad (1989), Rao (1991), Upadhyaya and Singh (1999), Singh (2003), Singh and Tailor (2003), Singh et al. (2004), Kadilar and Cingi (2004, 2006), Koyuncu and Kadilar (2009), Yan and Tian (2010), Subramani and Kkumarapandiyan (2012), Singh and Kumar (2012), Singh and Solanki (2013), Lu and Yan (2014), Sharma and Singh (2014). Although, plenty of estimators for population mean have been developed by many survey authors, rigorous work and experience in sampling theory gives ones scope of realization that still there remains possibilities of developing novel estimators with increased efficiency.

Ordinarily, a sampler assumes that prior information on auxiliary characters is complete and is available at our disposal prior to sampling. But one may encounter numerous realistic situations where the parametric value of population mean $\bar{X}$ is unknown. To cope up with the absence of prior auxiliary information, the use of double or two phase sampling technique is suggested. Besides its simplicity, double sampling technique confirms to be more flexible, powerful and considerably cost effective procedure to devise reliable estimates. Neyman (1938) was premier statistician who utilized double sampling technique for the purpose of stratification. This technique consists in first selecting a bigger sample of size n' from a universe of size N, while a sub- sample of size n generally known as second phase sample is chosen from n' to observe the characteristic under study. A preliminary large sample observed at first stage of this technique helps a sampler to estimate $\bar{X}$, thus using this technique several estimators of population mean has been formulated. The simplest estimator recorded in the literature is the usual biased estimator defined as

$$\hat{\bar{Y}}_{DR} = \frac{\bar{y}}{\bar{x}}\bar{x}' = \hat{R}\,\bar{x}' \tag{1.4}$$

The relative bias and sampling variance of $\hat{\bar{Y}}_{DR}$ is given by

$$Bias\left(\hat{\bar{Y}}_{DR}\right) = \left(\frac{1}{n} - \frac{1}{n'}\right)\left(C_X^2 - C_X C_Y \rho\right) \tag{1.5}$$

$$V\left(\hat{\bar{Y}}_{DR}\right) = \left(\frac{1}{n} - \frac{1}{n'}\right)S_Y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right)\left(S_Y^2 + R^2 S_X^2 - 2RS_{YX}\right)$$

Assuming N large, we can write

$$V\left(\hat{\bar{Y}}_{DR}\right) = \frac{S_Y^2}{n'} + \left(\frac{1}{n} - \frac{1}{n'}\right)\left(S_Y^2 + R^2 S_X^2 - 2RS_{YX}\right) \tag{1.6}$$

Where

$$R = \frac{\bar{Y}}{\bar{X}},\; S_Y^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^2$$

$$S_X^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2,\; S_{YX} = \frac{1}{n-1}\sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)\left(X_i - \bar{X}\right)$$

For a high positive correlation, the double sampling ratio estimator for estimating $\bar{Y}$ defined in Eq. (1.4) produced results with greater efficiency as compared to the results yielded by usual sample mean estimator. In this connection, strenuous efforts have been made to enhance the above double sampling ratio estimator by employing auxiliary information in the form of first two moments about zero.

II. THE PROPOSED ESTIMATOR

Let U denote finite population under investigation with $U_1, U_2, U_3 .... U_N$ as N distinct and identifiable units. Let y and x indicates main and ancillary variable and (Y_i, X_i) denotes the i^{th} unit of population where $i = 1,2,..... N$. Let $\bar{Y}$, $\bar{X}$ denote population mean and S_Y^2, S_X^2 denote population variance of main and ancillary characteristics respectively.

We have,

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i \quad \bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i \quad ,S_Y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^2$$

and

$$S_X^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2 \;,\; S_{YX} = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)\left(X_i - \bar{X}\right)$$

and $\rho = \frac{S_{yx}}{S_Y S_X}$ denotes population correlation coefficient between main and auxiliary variable.

Also

let $\mu_{rs} = \frac{1}{N}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^r\left(X_i - \bar{X}\right)^s$, $C_Y^2 = \frac{S_Y^2}{\bar{Y}^2}$, $C_X^2 = \frac{S_X^2}{\bar{X}^2}$

$$\lambda = \frac{\mu_{12}}{\bar{Y}\sigma_X^2} = \frac{\mu_{12}}{\bar{Y}\mu_{02}} \;,\; \beta_2 = \frac{\mu_{04}}{\mu_{02}^2} \;,\; \beta_1 = \frac{\mu_{03}^2}{\mu_{02}^3} \;,\; \gamma_1 = \sqrt{\beta_1}\,.$$

In double or two-phase sampling technique, using SRSWOR design in either phases,

1. Let $\left(x_1', x_2', ..., x_{n'}'\right)$ represents a preliminary large sample of size n' taken only on auxiliary variable at first phase. Also, let first phase sample mean on x be denoted as $\bar{x}'$.
2. Let $\left\{(y_1, x_1), (y_2, x_2), ..., (y_n, x_n)\right\}$ represents a sub sample of size n selected from the preliminary large sample of size n′ taken at first phase. This second phase sample is observed on both main variable under study and an associated auxiliary character. Also, let second phase sample mean on y and x be denoted as $\bar{y}, \bar{x}$ respectively.

We have,

$\bar{x}' = \frac{1}{n'}\sum_{i=1}^{n'} x'_i$, $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and

$\bar{\theta}_x = \frac{1}{n}\sum_{i=1}^{n} x_i^2$, $\bar{\theta}'_x = \frac{1}{n'}\sum_{i=1}^{n'} x_i'^2$, $\hat{R} = \frac{\bar{y}}{\bar{x}}$

where $\bar{x}$, $\bar{x}'$, $\bar{\theta}_x$ and $\bar{\theta}'_x$ represents first two moments about zero.

Motivated by the knowledge on first two moments about zero and using it as auxiliary information, we propose an enhanced double sampling ratio estimator $\hat{\hat{Y}}_{EDR}$ as

$$\hat{\hat{Y}}_{EDR} = \hat{R}\, g\left(\frac{\bar{x}}{\bar{x}'}, \frac{\bar{\theta}_x}{\bar{\theta}'_x}\right)\bar{x}' = \hat{R}\, g(u_1, u_2)\bar{x}' \tag{2.1}$$

where $g(u_1, u_2)$ satisfies the validity conditions of Taylor's series expansion and is also a bounded function of $t = (u_1,\ u_2)$ such that at point $T = (1,\ 1)$ we have

$$g(t = T) = 1 \tag{2.2}$$

We obtain first order partial derivatives as

1. $$\left.\frac{\partial}{\partial u_1} g(u_1, u_2)\right]_T = g_1 \tag{2.3}$$
2. $$\left.\frac{\partial}{\partial u_2} g(u_1, u_2)\right]_T = g_2 \tag{2.4}$$

And second order partial derivatives as

3. $$\left.\frac{\partial^2}{\partial u_1 \partial u_2} g(u_1, u_2)\right]_T = g_{12} \tag{2.5}$$
4. $$\left.\frac{\partial^2}{\partial u_1^2} g(u_1, u_2)\right]_T = g_{11} \tag{2.6}$$
5. $$\left.\frac{\partial^2}{\partial u_2^2} g(u_1, u_2)\right]_T = g_{22} \tag{2.7}$$

III. EXPRESSION FOR BIAS AND MEAN SQUARED ERROR (MSE) FOR THE PROPOSED RATIO ESTIMATOR

For studying the properties of suggested enhanced ratio estimator, we define

$$\bar{y} = \bar{Y}(1 + e_0) \qquad \bar{x} = \bar{X}(1 + e_1)$$
$$\bar{x}' = \bar{X}(1 + e'_1) \qquad \bar{\theta}_x = \bar{\theta}_X(1 + e_2)$$
$$\bar{\theta}'_x = \bar{\theta}_X(1 + e'_2) \tag{3.1}$$

Since population under consideration is large enough relative to sample, for simplicity we ignore finite population correction terms.

So that

$$E(e_i) = 0\ (i = 0,1,2) \text{ and } E(e'_j) = 0\ (j = 1,2) \tag{3.2}$$

$$E(e_0^2) = \frac{1}{n} C_Y^2$$
$$E(e_1^2) = \frac{1}{n} C_X^2$$
$$E(e_1'^2) = \frac{1}{n'} C_X^2$$
$$E(e_2^2) = \frac{1}{n\bar{\theta}_X^2}\left(\mu_{04} + 4\bar{X}\mu_{03} + 4\bar{X}^2\mu_{02} - \mu_{02}^2\right)$$
$$E(e_2'^2) = \frac{1}{n'\bar{\theta}_X^2}\left(\mu_{04} + 4\bar{X}\mu_{03} + 4\bar{X}^2\mu_{02} - \mu_{02}^2\right)$$
$$E(e_0 e_1) = \frac{1}{n}\rho C_Y C_X$$
$$E(e_0 e'_1) = \frac{1}{n'}\rho C_Y C_X$$
$$E(e_0 e_2) = \frac{1}{n\bar{Y}\bar{\theta}_X}\left(\mu_{12} + 2\bar{X}\mu_{11}\right)$$
$$E(e_0 e'_2) = \frac{1}{n'\bar{Y}\bar{\theta}_X}\left(\mu_{12} + 2\bar{X}\mu_{11}\right)$$
$$E(e_1 e'_1) = \frac{1}{n'} C_X^2$$
$$E(e_1 e_2) = \frac{1}{n\bar{X}\bar{\theta}_X}\left(\mu_{03} + 2\bar{X}\mu_{02}\right)$$
$$E(e_1 e'_2) = \frac{1}{n'\bar{X}\bar{\theta}_X}\left(\mu_{03} + 2\bar{X}\mu_{02}\right)$$
$$E(e'_1 e_2) = \frac{1}{n'\bar{X}\bar{\theta}_X}\left(\mu_{03} + 2\bar{X}\mu_{02}\right)$$
$$E(e'_1 e'_2) = \frac{1}{n'\bar{X}\bar{\theta}_X}\left(\mu_{03} + 2\bar{X}\mu_{02}\right)$$
$$E(e_2 e'_2) = \frac{1}{n'\bar{\theta}_X^2}\left(\mu_{04} + 4\bar{X}\mu_{03} + 4\bar{X}^2\mu_{02} - \mu_{02}^2\right) \tag{3.3}$$

In order to simplify we expand $g(u_1,\ u_2)$ about the point $T = (1,\ 1)$ using Taylor's series expansion. The result obtained to the first degree of approximation is given as

$$\hat{\hat{Y}}_{EDR} = \hat{R}\Big\{g(1,1) + (u_1 - 1)g_1 + (u_2 - 1)g_2$$
$$+ \frac{1}{2!}\left[(u_1 - 1)^2 g_{11} + (u_2 - 1)^2 g_{22} + 2(u_1 - 1)(u_2 - 1)g_{12}\right]$$
$$+ \frac{1}{3!}\left[\left\{(u_1 - 1)\frac{\partial}{\partial u_1} + (u_2 - 1)\frac{\partial}{\partial u_2}\right\}^3 g\left(u_1^*, u_2^*\right)\right]\Big\}\bar{x}' \tag{3.4}$$

Where first and second order partial derivatives $g_1, g_2, g_{11}, g_{22}, g_{12}$ are already defined and $u_1^* = 1 + h(u_1 - 1)$ and $u_2^* = 1 + h(u_2 - 1)$ for $0 < h < 1$.

Retaining only second order terms and rewriting above Eq. (3.4) in terms of $e_i's$, to the approximation of order one, we get

$$\hat{\hat{\bar{Y}}}_{EDR}-\bar{Y}=\bar{Y}\left(e_0-e_1+e_1'+e_1g_1-e_1'g_1+e_2g_2-e_2'g_2\right)$$
$$+\bar{Y}\left\{\left(e_0e_1-e_0e_1'-e_1^2+e_1e_1'\right)g_1\right.$$
$$+\left(e_2'^2-e_2e_2'+e_0e_2-e_0e_2'-e_1e_2+e_1e_2'+e_1'e_2-e_1'e_2'\right)g_2$$
$$+\frac{1}{2}\left(e_1^{\,2}+e_1'^{\,2}-2e_1e_1'\right)g_{11}$$
$$+\frac{1}{2}\left(e_2^{\,2}+e_2'^{\,2}-2e_2e_2'\right)g_{22}+\left(e_1e_2-e_1e_2'-e_1'e_2+e_1'e_2'\right)g_{12}$$
$$\left.+\left(e_1^2-e_1e_1'-e_0e_1+e_0e_1'\right)\right\} \qquad (3.5)$$

Further we take expectation on both the sides of Eq. (3.5) and incorporating above results, the bias of suggested enhanced double sampling ratio estimator $\hat{\hat{\bar{Y}}}_{EDR}$ up to terms of $O\left(n^{-1}\right)$ is expressed as

$$Bias\left(\hat{\hat{\bar{Y}}}_{EDR}\right)=\left\{E\left(\hat{\hat{\bar{Y}}}_{EDR}\right)-\bar{Y}\right\}$$
$$=\bar{Y}\left(\frac{1}{n}-\frac{1}{n'}\right)\left(C_X^2-\rho C_YC_X\right)+\bar{Y}\left(\frac{1}{n}-\frac{1}{n'}\right)\left\{\left(\rho C_YC_X-C_X^2\right)g_1\right.$$
$$+\left[\frac{1}{\bar{Y}\bar{\theta}_X}\left(\mu_{12}+2\bar{X}\mu_{11}\right)-\frac{1}{\bar{X}\bar{\theta}_X}\left(\mu_{03}+2\bar{X}\mu_{02}\right)\right]g_2$$
$$+\frac{1}{2}C_X^2g_{11}+\frac{1}{2\bar{\theta}_X^2}\left(\mu_{04}+4\bar{X}\mu_{03}+4\bar{X}^2\mu_{02}-\mu_{02}^2\right)g_{22}$$
$$\left.+\frac{1}{\bar{X}\bar{\theta}_X}\left(\mu_{03}+2\bar{X}\mu_{02}\right)g_{12}\right\} \qquad (3.6)$$

We now take square of Eq. (3.5) on both sides and after further simplifications and taking expectation, the MSE of the suggested ratio estimator is given as

$$MSE\left(\hat{\hat{\bar{Y}}}_{EDR}\right)=\left\{E\left(\hat{\hat{\bar{Y}}}_{EDR}\right)-\bar{Y}\right\}^2 \qquad (3.7)$$

On substituting expected values given in Eq. (3.2) to Eq. (3.3), the above Eq. (3.7) can be simplified as

$$MSE\left(\hat{\hat{\bar{Y}}}_{EDR}\right)$$
$$=\bar{Y}^2\left\{\left[E\left(e_0^2\right)+E\left(e_1^2\right)+E\left(e_1'^2\right)-2E\left(e_0e_1\right)+2E\left(e_0e_1'\right)-2E\left(e_1e_1'\right)\right]\right.$$
$$+g_1^2\left[E\left(e_1^2\right)+E\left(e_1'^2\right)-2E\left(e_1e_1'\right)\right]+g_2^2\left[E\left(e_2^2\right)+E\left(e_2'^2\right)-2E\left(e_2e_2'\right)\right]$$
$$+2g_1\left[E\left(e_0e_1\right)-E\left(e_0e_1'\right)-E\left(e_1^2\right)+E\left(e_1e_1'\right)+E\left(e_1e_1'\right)-E\left(e_1'^2\right)\right]$$
$$+2g_2\left[E\left(e_0e_2\right)-E\left(e_0e_2'\right)-E\left(e_1e_2\right)+E\left(e_1e_2'\right)+E\left(e_1'e_2\right)-E\left(e_1'e_2'\right)\right]$$
$$\left.+2g_1g_2\left[E\left(e_1e_2\right)-E\left(e_1e_2'\right)-E\left(e_1'e_2\right)-E\left(e_1'e_2'\right)\right]\right\}$$

$$MSE\left(\hat{\hat{\bar{Y}}}_{EDR}\right)=\frac{S_Y^2}{n'}+\left(\frac{1}{n}-\frac{1}{n'}\right)\left(S_Y^2+R^2S_X^2-2RS_{YX}\right)$$
$$+\bar{Y}^2\left(\frac{1}{n}-\frac{1}{n'}\right)\left\{C_X^2g_1^2+\frac{1}{\bar{\theta}_X^2}\left(\mu_{04}+4\bar{X}\mu_{03}+4\bar{X}^2\mu_{02}-\mu_{02}^2\right)g_2^2\right.$$
$$+2\left(\rho C_YC_X-C_X^2\right)g_1$$
$$+2\left[\frac{1}{\bar{Y}\bar{\theta}_X}\left(\mu_{12}+2\bar{X}\mu_{11}\right)-\frac{1}{\bar{X}\bar{\theta}_X}\left(\mu_{03}+2\bar{X}\mu_{02}\right)\right]g_2$$
$$\left.+2\frac{1}{\bar{X}\bar{\theta}_X}\left(\mu_{03}+2\bar{X}\mu_{02}\right)g_1g_2\right\} \qquad (3.8)$$

The value of $MSE\left(\hat{\hat{\bar{Y}}}_{EDR}\right)$ given in Eq. (3.8) depends on the values of g_1 and g_2, Hence we differentiate the above Eq. (3.8) with respect to g_1 and g_2. The obtained optimum values of g_1 and g_2 for which $MSE\left(\hat{\hat{\bar{Y}}}_{EDR}\right)$ in Eq. (3.8) attains the minimum value are

$$g_1=1-\frac{1}{C_X^2}\left[\rho C_YC_X-\frac{1}{\bar{Y}\Delta}\left(\mu_{03}+2\bar{X}\mu_{02}\right)\left(\delta_1-\delta_2\right)\right] \qquad (3.9)$$

$$g_2=-\frac{\bar{X}\bar{\theta}_X}{\bar{Y}\Delta}\left(\delta_1-\delta_2\right) \qquad (3.10)$$

where

$\delta_1=C_X^2\bar{X}\left(\mu_{12}+2\bar{X}\mu_{11}\right)$, $\delta_2=\rho C_YC_X\bar{Y}\left(\mu_{03}+2\bar{X}\mu_{02}\right)$ and

$$\Delta=C_X^2\bar{X}^2\left(\mu_{04}+4\bar{X}\mu_{03}+4\bar{X}^2\mu_{02}-\mu_{02}^2\right)-\left(\mu_{03}+2\bar{X}\mu_{02}\right)^2$$
$$=\mu_{02}^3\left(\beta_2-\beta_1-1\right)>0$$

In order to obtain the expression for minimum mean squared error of $\hat{\hat{\bar{Y}}}_{EDR}$ we now substitute the optimum values of g_1 and g_2 obtained above in Eq. (3.8), as a result we get

$$MSE\left(\hat{\hat{\bar{Y}}}_{EDR}\right)_{\min}=\frac{S_Y^2}{n'}+\left(\frac{1}{n}-\frac{1}{n'}\right)\left(S_Y^2+R^2S_X^2-2RS_{YX}\right)$$
$$-\bar{Y}^2\left(\frac{1}{n}-\frac{1}{n'}\right)\left[\rho^2C_Y^2+C_X^2-2\rho C_YC_X+\frac{\left(\delta_1-\delta_2\right)^2}{C_X^2\bar{Y}^2\Delta}\right] \qquad (3.11)$$

or

$$MSE\left(\hat{\hat{\bar{Y}}}_{EDR}\right)_{\min}=\frac{S_Y^2}{n'}+\left(\frac{1}{n}-\frac{1}{n'}\right)\left(S_Y^2+R^2S_X^2-2RS_{YX}\right)$$
$$-\left(\frac{1}{n}-\frac{1}{n'}\right)\left[\rho^2S_Y^2+R^2S_X^2-2\rho RS_YS_X+\frac{\bar{X}^2\left(\delta_1-\delta_2\right)^2}{S_X^2\Delta}\right] \qquad (3.12)$$

or

$$MSE\left(\hat{\hat{\bar{Y}}}_{EDR}\right)_{\min}=\frac{S_Y^2}{n'}+\left(\frac{1}{n}-\frac{1}{n'}\right)\left(S_Y^2+R^2S_X^2-2RS_{YX}\right)$$
$$-\left(\frac{1}{n}-\frac{1}{n'}\right)\left[\rho^2S_Y^2+R^2S_X^2-2RS_{YX}+\frac{\bar{X}^2\left(\delta_1-\delta_2\right)^2}{S_X^2\Delta}\right] \qquad (3.13)$$

IV. EFFICIENCY COMPARISON

In order to prove superiority of the suggested enhanced ratio estimator, let us consider following estimators for population mean

1. The general estimator in case of SRSWOR and its relative MSE is defined as

$\bar{y}_{wor} = \bar{y} = sample \;\; mean$

$$MSE(\bar{y}_{wor}) = \frac{\bar{Y}^2 C_y^2}{n} \quad (4.1)$$

2. The product estimator given by Murthy (1964) and its relative MSE is defined as

$$\bar{y}_p = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)$$

$$MSE(\bar{y}_p) = \frac{\bar{Y}^2}{n}\left(C_y^2 + C_x^2 + 2\rho C_y C_x\right) \quad (4.2)$$

3. Estimator given by Kadilar and Cingi (2003) and its relative MSE is defined as

$$\bar{y}_{kc} = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^2$$

$$MSE(\bar{y}_{kc}) = \frac{\bar{Y}^2}{n}\left(C_y^2 + 4C_x^2 - 4\rho C_y C_x\right) \quad (4.3)$$

4. The usual double sampling ratio estimator and its relative MSE is defined as

$$\hat{\bar{Y}}_{DR} = \frac{\bar{y}}{\bar{x}}\bar{x}' = \hat{R}\;\bar{x}'$$

$$MSE\left(\hat{\bar{Y}}_{DR}\right) = \frac{S_Y^2}{n'} + \left(\frac{1}{n} - \frac{1}{n'}\right)\left(S_Y^2 + R^2 S_X^2 - 2RS_{YX}\right) \quad (4.4)$$

On comparing equations Eq. (4.1), Eq. (4.2), Eq. (4.3) and Eq. (4.4) with Eq. (3.13) we can reasonably conclude that the suggested enhanced double sampling ratio estimator utilizing known values of first and second moment about zero has lesser MSE under the following conditions.

$$MSE(\bar{y}_{wor}) - MSE\left(\hat{\bar{Y}}_{EDR}\right)_{\min} \geq 0 \quad (4.6)$$

$$MSE(\bar{y}_p) - MSE\left(\hat{\bar{Y}}_{EDR}\right)_{\min} \geq 0 \quad (4.7)$$

$$MSE(\bar{y}_{kc}) - MSE\left(\hat{\bar{Y}}_{EDR}\right)_{\min} \geq 0 \quad (4.8)$$

$$MSE(\hat{\bar{Y}}_{DR}) - MSE\left(\hat{\bar{Y}}_{EDR}\right)_{\min} \geq 0 \quad (4.9)$$

Therefore, for obtaining precise results, the use of proposed double sampling ratio estimator under practical situation is strongly recommended.

V. EMPIRICAL STUDY

In the previous section, efficiency of proposed estimator was compared theoretically; however this section encompasses numerical illustrations to examine the performance and practical utility of suggested estimator. For this purpose two natural population data sets have been studied. Computation of the required values has been done and their respective description is given below.

Population I**:** Consider the data given in Singh and Chaudhary (1997, Page Number: 154-155)

y : Number of milch animals in survey

x : Number of milch animals in census

The computed population parameters are given in Table I below.

Table I: Population parameters for Population I

$\mu_{02}=$ 431.5847751	$\mu_{20}=$ 270.9134948	$\mu_{11}=$ 247.3944637	$\mu_{12}=$ 3119.839406
$\mu_{03}=$ 5789.778954	$\mu_{40}=$ 154027.4827	$\mu_{21}=$ 2422.297374	$\mu_{22}=$ 210594.3138
$\mu_{30}=$ 2273.46265	$\mu_{04}=$ 508642.4447	$\bar{y}=$ 1133.294118	$\bar{x}=$ 1140.058824
$S_x=$ 20.77461853	$S_y=$ 16.45945002	$\rho=$ 0.723505104	$\beta_2(y)=$ 2.098635139
$\beta_2(x)=$ 2.730740091	$C_X=$ 0.018222409	$C_Y=$ 0.014523547	$\beta=$ 0.573223334
$n=17$	$n'=30$		

Population II: Consider the data given in Mukhopadhyay (2012, Page Number: 104)

y : Quantity of raw materials (in lakhs)

x : Number of laborers (in thousands)

The computed population parameters are given in Table II below.

Table II: Population parameters for Population II

$\mu_{02}=$ 9704.4475	$\mu_{20}=90.95$	$\mu_{11}=$ 612.725	$\mu_{12}=$ 93756.3475
$\mu_{03}=$ 988621.5173	$\mu_{40}=$ 35456.4125	$\mu_{21}=$ 11087.635	$\mu_{22}=$ 2893630.349
$\mu_{30}=$ 1058.55	$\mu_{04}=$ 341222548.2	$\bar{y}=41.5$	$\bar{x}=441.95$
$S_x=$ 98.51115419	$S_y=$ 9.53677094	ρ =0.652197067	$\beta_2(y)=$ 4.286367314
$\beta_2(x)$ =3.623231573	$C_X=$ 0.22290113	C_Y =0.229801709	β =0.06313857 6
$n=20$	$n'=35$		

Table III below represents a summarized data statistics for both the population taken under consideration. The computed values of MSE and Percent Relative Efficiency (PRE) for different traditional estimators considered for comparison purposes have been calculated. Through empirical results, authors conclude that the proposed enhanced double sampling ratio estimator performs better under practical situations.

Table III: A summarized MSE and PRE

Estimat	Population	Population

ors	I		II	
	MSE	PRE	MSE	PRE
$\bar{y}_{wor}$	15.93609	129.33767	4.5475	122.29113
$\bar{y}_{p}$	69.95557	567.76105	14.57960	392.11705
$\bar{y}_{kc}$	58.41884	474.12867	10.15424	273.09744
$\hat{\bar{Y}}_{DR}$	14.26967	115.81298	3.91530	105.30177
$\hat{\bar{Y}}_{EDR}$	12.32130	100	3.71817	100

VI. CONCLUSION

This present study proposes an enhanced double sampling ratio estimator using knowledge on first and second moments about zero of an auxiliary character. The theoretical comparison of the formulated estimator on the basis of mean squared error is carried out with some existing estimators. From theoretical results and numerical illustrations it is reasonably concluded that the suggested ratio estimator have lesser mean square error value in contrast to the existing estimators. Hence it is analytically recognized that the suggested estimator has performed relatively better and achieved efficient results under PRE criterion. Therefore, based on the findings of this paper, we strongly recommend application of proposed estimator for the practical purposes against the existing estimators.

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