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Beta inverse Maxwell Distribution with its Statistical Properties and Applications

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Abstract:
The Maxwell distribution is one of the basic distributions in physics and commonly used to in statistical mechanics to determine the speed of molecules beside being popular in statistics for modeling life time data. In this article, we review the Beta Inverse Maxwell distribution and establish some statistical properties. We derive maximum likelihood estimator with confidence intervals.
Keywords: **Inverse Maxwell Distribution, Beta Inverse Maxwell distribution, Maximum likelihood estimator, Information Matrix, Confidence interval**

Introduction

Maxwell distribution (MWD) also known as the Maxwell Boltzmann distribution. It is named after James clark Maxwell and Ludwig Boltzmann. The distribution is commonly used in statistical mechanics to determine the speed of molecules. The distribution has been recognized as a lifetime models in survival analysis. Famous for its applicability in various streams of science such as astrophysics, chemistry and engineering. Among many, Tyagi and Bhattacharya (1989), Chaturvedi and Rani (1998), Bekker and Roux (2005) and Tomer and Panwar (2015) are a few expressive contributions that advocated the use of Maxwell distribution in life-testing experiments. The Maxwell distribution is defined as follows.

A random variable (rv) Y is said to follows MWD if its probability density function (pdf) is given by

$$f(y, \theta)=\frac{4y^2e^{-y^2/2}}{\sqrt{\pi}\theta^{3/2}} \qquad y>0, \theta>0 \tag{1}$$

Where θ is the scale parameter.
The mean and variance are given as

$$E(X) = 2\sqrt{\frac{\theta}{\pi}} \tag{2}$$

$$\text{Variance}=\frac{\theta(3\pi-8)}{2\pi} \tag{3}$$

If X has Maxwell distribution then the random variable $Y=\frac{1}{x}$ is said to follow inverse Maxwell distribution. The pdf of inverse Maxwell distribution is obtained by using the transformation

$$f(x,\theta)=f_x\left(\frac{1}{x},\theta\right)\left|\frac{dy}{dx}\right|$$

By using the

$$f(x, \theta)=\frac{4e^{-\frac{1}{\theta x^2}}}{\sqrt{\pi}\theta^{3/2}x^4} \qquad x>0, \theta>0 \tag{4}$$

The mean and variance of Maxwell distribution is

$$E(X) = \frac{2}{\sqrt{\pi\theta}}$$

$$Var(X) = \frac{2(\pi-2)}{\theta\pi}$$

The distribution given by (4) is called inverse Maxwell distribution. First time Singh and Srivastava (2014) briefly discussed about the inverse Maxwell distribution as a life time model.

Beta Inverse Maxwell Distribution: If $F_X(\theta)$ is the base cdf then a new distribution by using beta expression can be defined as

$$G_B(x,\theta,\alpha,\beta)=\frac{1}{B(\alpha,\beta)}\int_0^{Fx(\theta)} u^{\alpha-1}\,(1-u)^{\beta-1}du$$

Where B (….) is a beta function and two additional parameters $\alpha,\beta>0$ provides more flexible to the base model. The cdf of beta inverse Maxwell distribution is given by

$$G_{B\ inv\ MWD}(\text{X: α, β, θ})=\frac{\frac{2}{\sqrt{\pi}}\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right)}{\alpha B(\alpha,\beta)}F_1\left(\alpha,1-\beta,1+\alpha;\frac{2}{\sqrt{\pi}}\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right)\right) \quad (5)$$

Where $2F_1(a,b,c,z)=\sum_{n=0}^{\infty}\frac{\alpha_{nB_n}}{C_n}\frac{Z^n}{n!}(u)!=u(u+1\ldots\ldots\ldots\ldots..u+i-1)$ is well defined hyper geometric function. The pdf of beta inverse Maxwell distribution is.

$$g(\text{x: x:α,β,θ})=\frac{4e^{-\frac{1}{\theta x^2}}}{\text{B(α,β)}\sqrt{\pi}\theta^{3/2}\text{x}^4}\left[\frac{2}{\sqrt{\pi}}\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right)\right]^{\alpha-1}\left[1-\left(\frac{2}{\sqrt{\pi}}\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right)\right)^{\beta-1}\right] \quad (6)$$

Properties:

(i) If α=1, β =1 we get inverse Maxwell distribution

(ii) If we put $y=\frac{1}{x}$ we get Beta Maxwell distribution.

(iii) If α=1, β =1 and $y=\frac{1}{x^2}$ we get Gamma distribution $\left(\frac{1}{\theta},\frac{3}{2}\right)$.

Lemma 1: The limit of beta inverse Maxwell density as $x\rightarrow\infty\ is\ 0.$

$$\lim_{x\to\infty} g(x)=\lim_{x\to\infty}\frac{1}{B(\alpha,\beta)}\left[\frac{2}{\sqrt{\pi}}\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right)\right]^{\alpha-1}\text{X}\left[1-\left(\frac{2}{\sqrt{\pi}}\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right)\right)^{\beta-1}\right]\text{X}\frac{4e^{-x^2/2}}{\sqrt{\pi}\theta^{3/2}\text{x}^4}$$

$$=\lim_{x\to\infty}\frac{4e^{-x^2/2}}{\sqrt{\pi}\theta^{3/2}\text{x}^4}\lim_{x\to\infty}\frac{1}{B(\alpha,\beta)}\left[\frac{2}{\sqrt{\pi}}\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right)\right]^{\alpha-1}\lim_{x\to\infty}\left[1-\left(\frac{2}{\sqrt{\pi}}\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right)\right)^{\beta-1}\right.$$

$$=0$$

This is because $\lim_{x\to\infty}\frac{4e^{-\frac{1}{\theta x^2}}}{\sqrt{\pi}\theta^{3/2}\text{x}^4}=0$

The above indicate that the beta inverse Maxwell distribution has a mode.

Graphs of BIMD: The pdf, cdf, survival and hazard function for different combinations of parameters are given below.

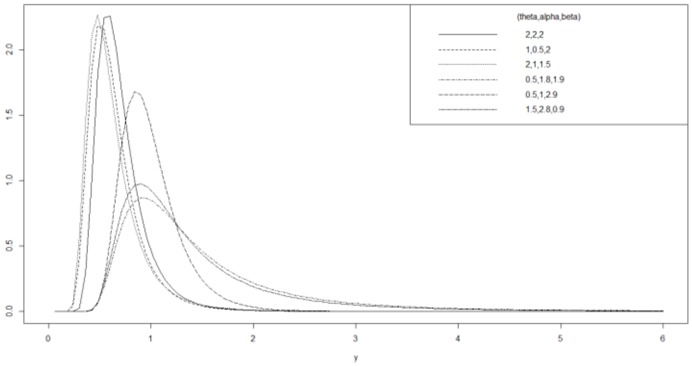

Figure 1: pdf of BIMD

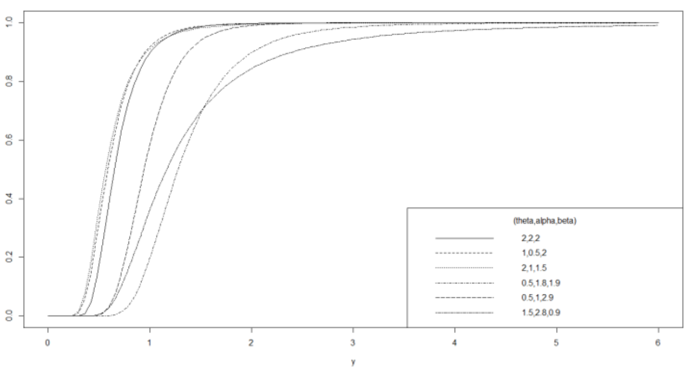

Figure 2 cdf of BIMD.

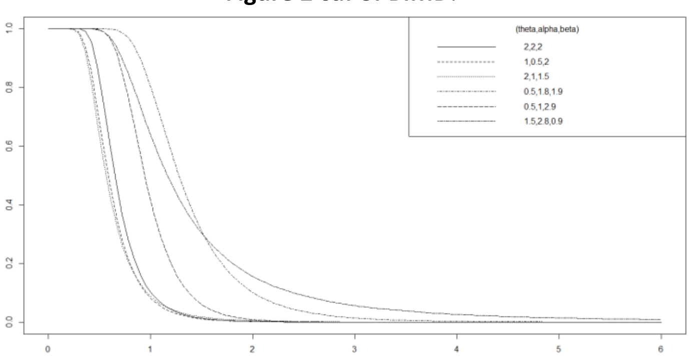

Figure 3: survival function of BIMD.

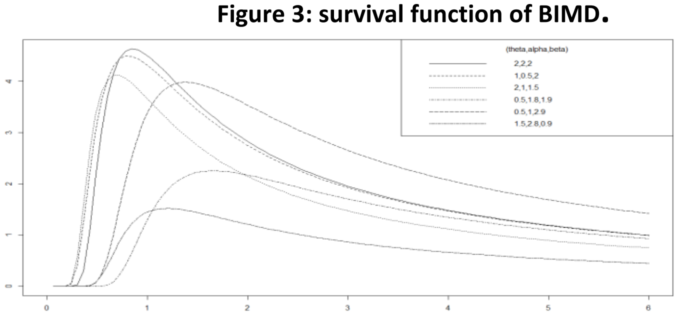

Figure 4 hazard rate

The expression for the hazard rate is not in closed form. But based on functional form we see that hazard rate of BIMD first increases and after attaining a maximum Value starts to decline. Hence it can be visualized from the figure (4) that it is of inverted U shape.

Random Number Generation for BIMD: For the simulation study purpose, we need random numbers. We can generate the random numbers from the BIMD by using inverse transform method. Algorithm for data generation is given below:

- Write the cdf of BIMD equal to u.
- Where u~U (0,1)
- Use inverse transform method, to find the value of yi in terms of u.
- $Y_i = \theta\,[\Gamma_{3/2}^{-1}\{\frac{\sqrt{\pi}}{2}B^{-1}(\mathrm{u},\alpha,\beta)\}]^{-1}$

  Where $\Gamma^{-1}(.)$ is inverted upper incomplete gamma function and B−1(.) is the inverted lower beta function.
- Use R function *Igamma.inv*(.) in zipfR package to generate random numbers from BIMD.

Parameter Estimation: the estimation of parameters **of** the beta inverse Maxwell distribution are obtain by the method of Maximum likelihood estimator

Taking likelihood of the equation (6) we get

$$L(x:\alpha,\beta,\theta)=\frac{4^n}{(\pi)^{\frac{n}{2}}B(\alpha,\beta)^n\prod_{i=1}^n x^4\theta^{3n/2}}\prod_{i=1}^n e^{-1/\theta x^2}\prod_{i=1}^n[\frac{2}{\sqrt{\pi}}\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right)]^{\alpha-1}\prod_{i=1}^n\left(1-(\frac{2}{\sqrt{\pi}}\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right))\right)^{\beta-1}$$

(4.4.1)

Taking log on both side

$$\log L(x:\alpha,\beta,\theta)=n\log 4-\frac{n}{2}\log\pi-n\log B(\alpha,\beta)\;-4\prod_{i=1}^n x_i-\frac{1}{\theta}\sum_{i=1}^n\frac{1}{xi}-\frac{3n}{2}\log\theta+(\alpha-1)\sum_{i=1}^n\log([\frac{2}{\sqrt{\pi}}\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right)]^{\alpha-1})+(\beta-1)\sum_{i=1}^n\log([1-\frac{2}{\sqrt{\pi}}\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right)]^{\beta-1})\quad(4.4.2)$$

Differentiating the above equation with respect to α, β and θ respectively and equating the result to zero we have

$$\frac{\partial logL(x:\alpha,\beta,\theta)}{\partial\alpha}=-n^{\phi}(\alpha)+n^{\phi}(\alpha+\beta)+\sum_{i=1}^n log\left(\frac{2}{\sqrt{\pi}}\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right)\right)=0$$

$$\frac{\partial logL(x:\alpha,\beta,\theta)}{\partial\beta}=-n^{\phi}(\beta)+n^{\phi}(\alpha+\beta)+\sum_{i=1}^n log\left(1-(\frac{2}{\sqrt{\pi}}\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right))\right)$$

$$\frac{\partial logL(x:\alpha,\beta,\theta)}{\partial\theta}=\frac{-3n}{2\theta}+\frac{1}{\theta^2}\sum_{i=1}^n\frac{1}{x}-(\alpha-1)\sum_{i=1}^n\frac{e^{\frac{-1}{\theta x^2}}}{x^3\theta^{5/2\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right)}}+(\beta-1)\sum_{i=1}^n\frac{2e^{\frac{-1}{\theta x^2}}}{\sqrt{\pi}x^3\theta^{5/2\left((1-(\frac{2}{\sqrt{\pi}}\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right))\right)}}$$

Where ϕ(k) is the logarithmic derivative of the gamma function. Solving the above equation simultaneously for α, β and θ gives us their respectively estimates. Also find the derivative of these equations gives us the diagonal elements of the Fisher information matrix. They are as follow.

$$\frac{\partial^2 logL(x:\alpha,\beta,\theta)}{\partial\alpha^2}=n^{\Phi 1}(\alpha)+n^{\Phi 1}(\alpha+\beta)$$

$$\frac{\partial^2 logL(x:\alpha,\beta,\theta)}{\partial\alpha\partial\beta}=n^{\Phi 1}(\alpha+\beta)=\frac{\partial^2 logL(x:\alpha,\beta,\theta)}{\partial\beta\partial\alpha}$$

$$\frac{\partial^2 logL(x:\alpha,\beta,\theta)}{\partial\alpha\partial\theta}=-\sum_{i=1}^n\frac{e^{\frac{-1}{\theta x^2}}}{x^3\theta^{5/2\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right)}}$$

$$\frac{\partial^2 logL(x:\alpha,\beta,\theta)}{\partial\theta^2}=\frac{3n}{2\theta^2}-\frac{2}{\theta^3}\sum_{i=1}^n\frac{1}{x}-(\alpha-1)\sum_{i=1}^n\frac{e^{\frac{-2}{\theta x^2}}}{x^6\theta^{5/2(\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right))^2}}+(\alpha-1)\sum_{i=1}^n\frac{5e^{\frac{-1}{\theta x^2}}}{2x^3\theta^{5/2\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right)}}$$

$$--(\alpha-1)\sum_{i=1}^n\frac{e^{\frac{-1}{\theta x^2}}}{x^5\theta^{9/2\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right)}}+(\beta-1)\sum_{i=1}^n\frac{4e^{\frac{-2}{\theta x^2}}}{\pi x^6\theta^{11/2\left((1-(\frac{2}{\sqrt{\pi}}\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right))\right)^2}}-$$

$$(\beta-1)\sum_{i=1}^n\frac{5e^{\frac{-1}{\theta x^2}}}{\sqrt{\pi}x^6\theta^{7/2\left((1-(\frac{2}{\sqrt{\pi}}\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right))\right)}}-(\beta-1)\sum_{i=1}^n\frac{2e^{\frac{-1}{\theta x^2}}}{\sqrt{\pi}x^8\theta^{9/2\left((1-(\frac{2}{\sqrt{\pi}}\Upsilon\left(\frac{3}{2},\frac{1}{\theta x^2}\right))\right)}}$$

Variance-Covariance Matrix for BIMD: The observed information matrix for the parameter is given by inverting the matrix made by the second derivative of normal equations. Therefore, we get the approximate Fisher's Information matrix which is given by

$$I\,(€^{-1})=\begin{bmatrix} I\theta\theta & I\theta\alpha & I\theta\beta\\ I\alpha\theta & I\alpha\alpha & I\alpha\beta\\ I\beta\theta & I\beta\alpha & I\beta\beta\end{bmatrix}$$

Where $\hat{€}=(\hat{\alpha},\hat{\theta},\hat{\beta})$ is the corresponding MLE of €, and.

$$I_{\alpha\alpha}=-\frac{\partial^2 l}{\partial\alpha^2},\quad I_{\alpha\theta}=-\frac{\partial^2 l}{\partial\alpha\partial\theta},\quad I_{\theta\alpha}=-\frac{\partial^2 l}{\partial\theta\partial\alpha},\quad I_{\theta\theta}=-\frac{\partial^2 l}{\partial\theta^2}$$

$I_{\beta\beta}$= - $\frac{\partial^2 I}{\partial \beta^2}$ $\quad I_{\alpha\theta}$= $\frac{\partial^2 I}{\partial \alpha \partial \beta}$ $\quad I_{\alpha\beta}$= - $\frac{\partial^2 I}{\partial \alpha \partial \beta}$ $\quad I_{\beta\alpha}$= - $\frac{\partial^2 I}{\partial \beta \partial \alpha}$

The approximate (observed) asymptotic variance-covariance matrix for the MLE of parameters θ, α and β can be found by inverting I ($\hat{\xi}$)

$$I^{-1}(\hat{\xi}) = \begin{bmatrix} var(\hat{\boldsymbol{\theta}}) & var(\hat{\boldsymbol{\theta}},\hat{\boldsymbol{\alpha}}) & var(\hat{\boldsymbol{\theta}},\hat{\boldsymbol{\beta}}) \\ var(\hat{\boldsymbol{\alpha}},\hat{\boldsymbol{\theta}}) & var(\hat{\boldsymbol{\alpha}}) & var(\hat{\boldsymbol{\alpha}},\hat{\boldsymbol{\beta}}) \\ var(\hat{\boldsymbol{\beta}},\hat{\boldsymbol{\theta}}) & var(\hat{\boldsymbol{\beta}},\hat{\boldsymbol{\theta}}) & var(\hat{\boldsymbol{\beta}}) \end{bmatrix}$$

Thus, using above equation we get the $100(1-\alpha)\%$ confidence limits for $\hat{\theta}$, $\hat{\beta}$ *and* $\hat{\alpha}$ is given by $\hat{\boldsymbol{\theta}} \mp Z_{\frac{\alpha}{2}}\sqrt{var(\widehat{\boldsymbol{\theta}})}$, $\hat{\boldsymbol{\alpha}} \mp Z_{\frac{\alpha}{2}}\sqrt{var(\widehat{\boldsymbol{\alpha}})}$ and $\hat{\boldsymbol{\beta}} \mp Z_{\frac{\alpha}{2}}\sqrt{var(\widehat{\boldsymbol{\beta}})}$ where $Z_{(\frac{\alpha}{2})}$ is upper $100(\frac{\alpha}{2})^{th}$ percentile of standard normal variate. The required calculations are given below

$$I_{\theta\theta}= \frac{-3n}{2\theta^2}+\frac{2}{\theta^3}\sum_{i=1}^{n}\frac{1}{x^2} - (\alpha-1)\sum_{i=1}^{n}\xi(xi,\theta) +(\beta-1)\sum_{i=1}^{n}\xi(xi,\theta)$$

$$I_{\alpha\alpha}= n\psi^1(\alpha) - n\psi^1(\alpha+\beta)$$

$$I_{\beta\beta}= n\psi^1(\beta) - n\psi^1(\alpha+\beta)$$

$$I_{\alpha\theta} = I_{\theta\alpha} = -\frac{1}{\theta^{5/2}}\sum_{i=1}^{n}\frac{xi^{-3}e^{\frac{-1}{\theta x^2}}}{\Gamma(\frac{3}{2},\frac{1}{\theta x^2})}$$

$$I_{\beta\theta} = I_{\theta\beta} = \frac{1}{\theta^{5/2}}\sum_{i=1}^{n}\frac{xi^{-3}e^{\frac{-1}{\theta x^2}}}{\Gamma(\frac{3}{2},\frac{1}{\theta x^2})}$$

$$I_{\alpha\beta} = I_{\beta\alpha} = - n\psi^1(\alpha+\beta)$$

where, $\psi^1(.)$ denotes the derivative of digamma function (known as trigamma function). Also, we have.

$$\zeta(xi,\theta) = \chi(xi,\theta)\left\{\frac{1}{\theta^2 xi^2}-\frac{5}{2\theta} - \chi(xi,\theta)\right\}$$

$$\chi(x_i,\theta)=\frac{e^{\frac{1}{-\theta x^2}}}{x^3}\left[\theta^{\frac{5}{2}}\Gamma\left(\frac{3}{2},\frac{1}{\theta x_i^2}\right)\right]^{-1}$$

$$\zeta1(xi,\theta) =\chi1(xi,\theta)\left\{\frac{1}{\theta^2 xi^2}-\frac{5}{2\theta} + \chi1(xi,\theta)\right\}$$

$$\chi1(xi,\theta)=\frac{e^{\frac{1}{-\theta x^2}}}{x^3}\left[\theta^{\frac{5}{2}}\gamma\left(\frac{3}{2},\frac{1}{\theta x_i^2}\right)\right]^{-1}$$

Simulation Study: We have done simulated study for different sample sizes by taking the initial parameter value of θ =1:2, α = 0:95 and β= 0:97. We have generated data using inverse transform method from BIMD and computed MSE and Absolute bias for the parameter estimates θ, α and β. We repeat the process for N=1000 times to get the average estimates. Simulated Data table for MSE and absolute average Bias for BIMD for different sample sizes is given Below in table 1. The calculations for CIs under different sample sizes for BIMD is given in table 2

Table 1: Average MSE and Absolute Bias for estimates under different sample sizes for simulated data for BIMD.

N		$\hat{\theta}$	$\hat{\boldsymbol{\alpha}}$	$\hat{\boldsymbol{\beta}}$**.**
20	MSE	0.7410 0.8551	0.5401 0.7320	0.0446 0.1795
30	MSE	0.7382 0.8537	0.5434 0.7345	0.04115 0.1732
50	MSE	0.7466 0.8607	0.5509 0.7402	0.0362 0.1623

Table 2: Interval estimates, length and shape for ACI with varying sample sizes for BIMD.

	n=20			n=30			n=50		
	$\hat{\boldsymbol{\theta}}$	$\hat{\boldsymbol{\alpha}}$	$\hat{\boldsymbol{\beta}}$	$\hat{\boldsymbol{\theta}}$	$\hat{\boldsymbol{\alpha}}$	$\hat{\boldsymbol{\beta}}$	$\hat{\boldsymbol{\theta}}$	$\hat{\boldsymbol{\alpha}}$	$\hat{\boldsymbol{\beta}}$
Interval estimation	0,1.0997	0,1.0997	0,0.7519	0.1760,2.1083	0,0.977	0,0.655	0.3386,1.934	0,0.54	0.5250,1.17316
Length	1.5095	1.0677	1.9322	1.2621	0.8796	1.5962	0.9565	0.662	1.2066

Real Data Application: The data set corresponding to lifetimes (in years) of retired women with temporary disabilities, which are incorporated in the Mexican insurance public system and who died during 2004. The data points are given by: 22, 24, 25, 27, 28, 29, 30, 31, 32, 33,34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 71, 74, 75, 79, 86. The dataset is scaled by diving 10. For the fitting of data, we have plotted ECDF and QQ plot which given in _figures 11 and 12. Also we have applied Anderson darling test to check the fitness of data. In AD test, the test statistic comes out to be 0.8520, along with the p value 0.4443, which is greater than 0.05, hence our data fits well to BIMD.

The point estimates for BIMD for real data (2) along with CIs are given below.

Table 3: Parameter estimates along with standard error and interval estimates for lifetime data for BIM

	$\hat{\theta}$	$\hat{\alpha}$	$\hat{\beta}$
Estimates	0.0121	0.2580	1.2948
		CIs	
SE	0.0062	0.1578	0.3280
Interval Est.	0,0.0243	0,0.5673	0.6518,1.9377
Length	0.0244	0.6187	1.2859

Results and Findings: From Simulation study in case of BIMD, we can see that average MSE, absolute bias and length decreases as the sample sizes increases. The results show that parameter Estimates are consistent. The interval Estimates contain the true parameter values both in case of CIs and boot-p methods. For BIMD, we see that average bias and MSE decreases as sample size increases. Also, results are satisfactory for interval estimates both in case of simulation and real data analysis.

Future Scope: In future the main interest will be applied part of EIMD and BIMD in different Felds. We may proceed with Bayesian analysis, under different priors to find Bayes estimates. Another fled may be to deal EIMD with different censoring schemes as per real life data Problems. Also, one can go with record value estimation problems, where hazard rate is Inverted bathtub shaped.

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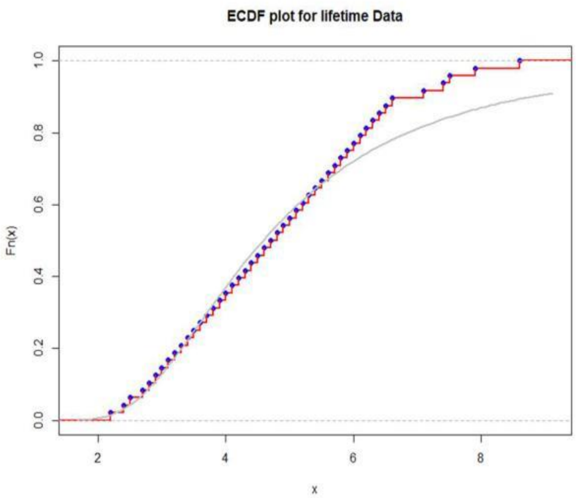
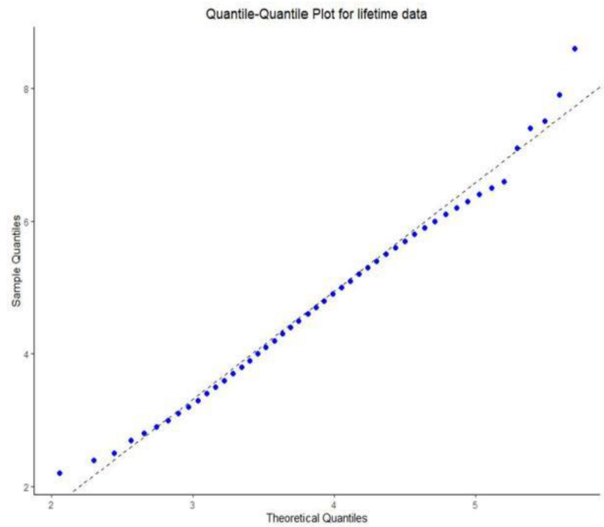
Appendix

QQ plot for GAGUrine data for BIMD.





ECDF plot for lifetime data for BIMD.
QQ plot for lifetime data for BIMD.

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