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Transient Analysis for a Multiple Vacations Queueing Model with Impatient Customers

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Abstract: **Present paper studies the effect of balking and reneging on a two-dimensional state queueing model with multiple vacations by considering the probability of exactly a-arrivals and b-services occurred by the time 't'. In addition, the system begins with some arbitrary number of clienteles (for e.g. there are always some numbers of clientele available before the opening of the ticket booking counter). "An arriving clientele may balk (do not enter) with a probability or renege according to the exponential distribution". The service times, vacation times, and inter-arrival times are negative exponentially distributed. We derived the time-dependent probabilities by using Laplace transformations and obtain some measurable outcomes of the system with the assistance of maple software. Validation of the model has been done with the existing results. Finally, an expected cost function is discussed.**

Index Terms: **Balking, Laplace Transform, Markovian queueing system, Multiple vacations, Reneging, Two-dimensional state model.**

I. INTRODUCTION

Queueing system with impatient customers have attracted many authors due to their extensive application in practical situations: hospital emergency rooms, handling critical patients, inventory systems that store perishable goods etc. In queues; "balking and reneging are common phenomena, as a consequence the customer either decides to join the queue or depart after joining the queue without getting service due to impatience". Performance Analysis of queueing systems with impatient customers in practical life problems is very helpful. Markovian queue with customers impatient customers have been discussed in Haight (1957) (1959) respectively. Anker & Gafarian (1963) (1964) has been first to study the integrate effect of impatient customers respectively. Abou-El- Ata (1991) derived probabilities for single server queueing system with impatient customers. Choudhury and Medhi (2011) investigated some aspects of balking and reneging in finite buffer queue. Rakesh and Sumeet (2012) calculated steady state probabilities for finite buffer markovian queueing model with balking and retention of reneged customers. Sharma & Indra (2020) obtained time independent probabilities for a single server markovian queueing model with reneging.

From the past few decades, Vacations Queueing system has attracted much attention from numerous researchers. Vacation Models was first talked about Cooper (1970). "Vacation:when the server finishes serving a unit and finds the system empty, however, it goes away for a length of time called a vacation" (1970). "In multiple vacations policy, server keeps on taking vacations until it finds at least one customer waiting in the system at the instant of vacation completion". Choudhary (2000) obtained probabilities for markovian queueing system with multiple vacations policy. Altman and Yechailli (2006) analyzed both single and multiple vacations for finite server queue. Later on a wide variety of work on multiple vacations can be found in Kelison & Servi (1987), Laxmi & Jyothsna (2014), Ammar (2015) and so on.

Queues comprising Balking, reneging and multiple vacations have attracted numerous researchers. Yue et al. (2006) had analyzed finite buffer markovian queueing model with balking, reneging, and server vacations. Ke & Wu (2012) obtained time independent probabilities for a multiserver machine repair model using matrix analytic method. Shinde and Patankar (2012) studied the state dependent bulk service queue with impatient customers and server vacation. Vijaya Laxmi et al. (2013) analyzed finite buffer markovian queueing model with balking and reneging.

This study considers a single server two dimensional state markovian queueing model with impatient customers and multiple vacations, considering initially “i” customers in the system. The two dimensional concept helps us to understand the probability of exactly n- arrivals and k- services occurs over a time interval of length t. Initially “i” concept makes this model finds its applicability in real world congestion problem for e.g. Call centers: Call arriving to a call centers are managed by agent to answer their calls. Primary calls are automatically answered by machines. The behaviour of the call may depend on several circumstances included waiting time and others. Each individual call may decide to balk or wait for some time and customer may abandon their call when their patience time expires. Further we put emphasis on time dependent probabilities because time independent probabilities do not reveal the actual picture of system under consideration. Various key measures have been discussed. Numerical results are calculated to display the impact of system parameters on performance measures. Finally we formulate a cost model to determine the Expected cost for the system**.**

II. Assumptions and Notations

1. “Arrivals follow a Poisson distribution with parameter λ”.
2. “Service times and vacation times are exponentially distributed with parameter μ and w respectively”.
3. “Inter-arrival times, service times and others involved in the system are statistically not dependent”.
4. “On arrival a customer either decides to join the queue with probability.
   β = prob{a unit joins the queue} or balk with probability (1-β) where $0 \leq \beta < 1$”
5. “Each customer upon joining the queue will wait a certain length of time for his service to begin. If it does not begin by then, he will get impatient (reneged) and may leave the queue without getting service. The reneging times follow exponential distribution with parameter ξ”.
6. Initially there are ‘i’ customers present at time t=0
   $$P_{i,0}(i,0) = 1$$
7. “The system state is given by (n, k), where n is the number of arrival and k is the number of departure up to time t”

$$P(i,0) = \sum_{k=0}^{\infty} P_{i+k,k,V}(i,0) = 1 \qquad (1)$$

III. Two-Dimensional STATE MODEL

A. Definitions

"$P_{n,k,B}(r,t)$ = The probability of exactly n arrivals, k departures and r- customers remain in the system by time t and the server is busy corresponding to the queue k< n”.

"$P_{n,k,V}(r,t)$ = The probability of exactly n arrivals, k departures and r- customers remain in the system by time t and the server is on vacation k ≤ n”.

"$P_{n,k}(r,t)$= The probability that there are exactly n arrivals and k departures and r- customers remain in the system by time t k ≤ n”.

B. Following are the equations describing the system

$$\frac{d}{dt}P_{n,k,V}(r,t) = -(\lambda\beta + w)P_{n,k,V}(r,t) + (\lambda\beta)P_{n-1,k,V}(r,t)$$
$$;\ 0 \leq k < n, r \geq 1 \qquad (4)$$

$$\frac{d}{dt}P_{n,n,V}(0,t) = -(\lambda\beta)P_{n,n,V}(0,t) + \mu P_{n,n-1,B}(1,t)\left(1-\delta_{n,0}\right)$$
$$;\ n \geq 0 \qquad (5)$$

$$\begin{aligned}\frac{d}{dt}P_{n,k,B}(r,t) = &-(\lambda\beta + \mu + (r-1)\xi)P_{n,k,B}(r,t) \\ &+ \lambda\beta P_{n-1,k,B}(r-1,t)\left(1-\delta_{k,n-1}\right) \\ &+ wP_{n,k,V}(r,t) + (\mu + r\xi)P_{n,k-1,B}(r+1,t)\end{aligned}$$
$$;0 \leq k < n, r \geq 1 \qquad (6)$$

Clearly,

$$P_{n,k}(r,t) = P_{n,k,V}(r,t) + P_{n,k,B}(r,t)(1-\delta_{(n,k)})$$
$$;\ n \geq k \geq 0 \qquad (7)$$

.

IV. Findings of Equation

Solving above equations with the help of Laplace transforms

$$\overline{P}_{n,k,V}(0,s) = \frac{1}{(s+\lambda\beta)}\delta_{(i,0)}P_{0,0,V}(0,0)$$
$$n = 0 = k \qquad (8)$$

$$\begin{aligned}\overline{P}_{n,0,V}(n,s) = &(\lambda\beta)^n \overline{H}_{n,1,0}^{\lambda\beta+w,\lambda,0}(s)\delta_{(i,0)} P_{0,0,V}(0,0) \\ &+ \sum_{x=1}^{\infty}(\lambda\beta)^{n-x}\, \overline{H}_{n-x+1,0,0}^{\lambda\beta+w,0,0}(s)\delta_{(i,x)}\, P_{x,0,V}(x,0)\end{aligned}$$
$$n > 0 \qquad (9)$$

$$\overline{P}_{n,0,B}(n,s) = w.\sum_{y=1}^{\infty}\frac{(\lambda\beta)^{n-y}}{\prod_{x=0}^{n-y}\{s+\lambda\beta+\mu+(n-x-1).\xi\}}\delta_{(i,0)}\ \overline{P}_{y,0,V}(y,s)$$

$$n > 0 \qquad (10)$$

$$\overline{P}_{n,k,v}(r,s) = (\lambda\beta)^{n-k}\mu\,\overline{H}_{n-k,1,0}^{\lambda\beta+w,\lambda\beta,0}(s)\overline{P}_{k,k-1,B}(1,s) + (\lambda\beta)^{n-k}\overline{H}_{n-k,1,0}^{\lambda\beta+w,\lambda\beta,0}(s)\,\delta_{(i,0)}P_{k,k,v}(0,0) + \sum_{x=k+1}^{\infty}(\lambda\beta)^{n-x}\overline{H}_{n-x+1,0,0}^{\lambda\beta+w,0,0}(s)\,\delta_{(i,x-k)}P_{x,k,v}(x-k,0)$$

$$n > k \geq 0 \qquad (11)$$

$$"\overline{P}_{n,k,B}(r,s) = \sum_{y=k+1}^{\infty}\frac{(\lambda\beta)^{n-y}.\{\mu+(y-k).\xi\}}{\prod_{x=0}^{n-y}\{s+\lambda\beta+\mu+(n-x-k-1).\xi\}}\overline{P}_{y,k-1,B}(y-k+1,s) + w.\sum_{y=k+1}^{\infty}\frac{(\lambda\beta)^{n-y}.}{\prod_{x=0}^{n-y}\{s+\lambda\beta+\mu+(n-x-k-1).\xi\}}\overline{P}_{y,k,V}(y-k,s)"$$

$$n > k \geq 0 \qquad (12)$$

$$\overline{P}_{n,n,v}(0,s) = \frac{\mu}{(s+\lambda\beta)}\overline{P}_{n,n-1,B}(1,s) + \frac{1}{(s+\lambda\beta)}\delta_{(i,0)}P_{n,n,v}(0,0)$$

$$n > 0 \qquad (13)$$

V. SUBSTANTIATIONS OF THE MODEL

Using Laplace Transform for $P_{n.}(r,t)$ we have:

1. $\bar{P}_{n.}(s) = \sum_{k=0}^{n}\left[(\bar{P}_{n,k,V}(r,s) + \bar{P}_{n,k,B}(r,s)(1-\delta_{n,k})\right]$

$$\bar{P}_{n.}(s) = \sum_{k=0}^{n}\bar{P}_{n,k}(r,s) = \frac{(\lambda\beta)^n}{(s+\lambda\beta)^{n+1}}$$

And its Inverse Laplace transform is

$$P_{n.}(r,t) = \frac{e^{-\lambda\beta t}(\lambda\beta t)^n}{n!}$$

2. $\sum_{n=0}^{\infty}\sum_{k=0}^{n}\{\bar{P}_{n,k,V}(r,s) + \bar{P}_{(n,k,B}(r,s)(1-\delta_{(n,k)})\} = \frac{1}{s}$

$$\sum_{n=0}^{\infty}\sum_{k=0}^{n}\{P_{n,k,V}(r,t)\delta_{(i,r)} + P_{n,k,B}(r,t)\delta_{(i,r)}(1-\delta_{(n,k)})\} = 1$$

VI. NUMERICAL RESULTS

1. The Table-I signifies the computative results for the following equations i.e. :

(a) $\sum_{k=0}^{n}P_{n,k}(r,t)$

(b) $\sum_{k=0}^{n}P_{n,k,B}(r,t)$

(c) $\sum_{k=0}^{n}P_{n,k,V}(r,t)$

Pr {n arrivals in (0, t)} = $\frac{e^{-\lambda t}(\lambda t)^n}{n!}$ = $\sum_{k=0}^{n}P_{n,k}(r,t)$ $= P_{n,.}(r,t)$

Table1: For Exactly n customers served by time t

λ	μ	w	β	ξ	T	n	$\frac{e^{-\lambda t}*(\lambda t)^n}{n!}$	$\sum_{k=0}^{n}P_{n,k,v}(r,t)$	$\sum_{k=0}^{n}P_{n,k,B}(r,t)$	$\sum_{k=0}^{n}P_{n,k}(r,t)$
1	2	1	1	1	3	1	0.149361	0.126884	0.022476	0.149361
1	2	1	1	1	3	3	0.224042	0.137282	0.086759	0.224041
1	2	1	1	1	3	5	0.100819	0.043792	0.057026	0.100818
2	2	1	1	1	3	1	0.014873	0.012634	0.002238	0.014872
2	2	1	1	1	3	3	0.089235	0.057204	0.032030	0.089235
2	2	1	1	1	3	5	0.160623	0.079041	0.081582	0.160623
1	2	1	1	1	4	1	0.073263	0.064437	0.008825	0.073262
1	2	1	1	1	4	3	0.195367	0.133144	0.062222	0.195366

1	2	1	1	1	4	5	0.156293	0.081579	0.074713	0.156293
2	2	1	1	1	4	1	0.002684	0.002360	0.000323	0.002683
2	2	1	1	1	4	3	0.028626	0.020172	0.008453	0.028626
2	2	1	1	1	4	5	0.091604	0.052652	0.038951	0.091603
2	4	1	1	1	4	5	0.091604	0.066676	0.024927	0.091603
1	2	1	1	1	4	4	0.195367	0.116786	0.078580	0.195366
1	2	1	1	1	3	6	0.050409	0.018292	0.032117	0.050409
3	2	1	1	1	3	1	0.0011106	0.000943	0.000167	0.001110
3	2	1	1	1	3	3	0.0149942	0.009920	0.005073	0.014994
3	2	1	1	1	3	5	0.0607268	0.032397	0.028329	0.060726

The sum of probabilities obtained for n- customers coincides with the numerical results given by "Pegden & Rosenshine [1982]"

1. The probability that exactly k number of customers have been served "when the server is on vacation i.e. $\sum_{n=k}^{\infty} P_{n.k,V}(r,t)$, when server is busy i.e. $\sum_{n=k}^{\infty} P_{n.k,B}(r,t)$ are computed for different sets of parameters and based on the following relationship $P_{.k}(r,t) = \sum_{n=k}^{\infty} P_{n,k}(r,t)$" where $P_{n,k}(r,t)$ is defined in Eq (7).

Table-II: for exactly k customers served by time t

λ=1,μ=4,w=1,i=0,β=1,ξ=1,k=0to6 "$P_{.k}(r,t) = P_{.k,B}(r,t) + P_{.k,V}(r,t)$"				
t=1	t=3	t=5	t=7	t=10
.81267	.229241	.04337217	.0068178	.000364553
.14489	.238016	.09076465	.0228947	.002047436
.03509	.220294	.14530303	.0525335	.006952801
.00627	.152513	.17159309	.0876024	.016758736
.00086	.082670	.15655165	.1115662	.030392408
.00009	.034542	.10809203	.1061810	.040640908
.00006	.009211	.04650679	.0621151	.032984574
.99991	**.966491**	**.7621834**	**.4497110**	**.13014141**

Table II coincides with table I of Hubbard et al. [1986]

Table III: For Exactly N customers in the system

λ=1,μ=2,w=1,i=1,β=1,ξ=1,N=0to6 $P_N(t) = P_B(N,t) + P_v(N,t)$				
Time(t) =1	Time(t) =2	Time(t) =3	Time(t)=4	Time(t) =5
.20143	.278098	.2914123	.275931	.237900
.36639	.293803	.2854374	.255303	.204680
.26896	.209882	.1807455	.147317	.107949
.11708	.118871	.0922748	.066284	.043663
.03573	.055176	.0419344	.026383	.015244
.00831	.021106	.0176731	.010168	.004983
.0015	.006498	.0066042	.003741	.001539
.99942	**.983436**	**.916082**	**.78513**	**.615961**

Table-IV:

λ=1,μ=2,w=1,i=1,β=0.2 ,ξ=0.2			
	$\sum_{n=0}^{\infty}\sum_{k=0}^{n} P_{i,j,v}(r,t)$	$\sum_{n=0}^{\infty}\sum_{k=0}^{n} P_{i,j,B}(r,t)$	total
t=1	.736882	.263061	**.999943**
t=2	.810868	.18835	**.999218**
t=3	.861457	.135185	**.996642**
t=4	.881595	.109325	**.99092**
t=5	.884399	.096613	**.981012**

2. "The Probability of exactly N customers in the system at time t, is denoted by $P_N(t)$ and it can be expressed in terms of $P_{n,k}(r,t)$ as written below"

$$"P_N(t) = \sum_{k=0}^{\infty} P_{k+N,k}(N,t)"$$

$$"P_N(t) = P_B(N,t) + P_v(N,t)"$$

Where, $"P_B(N,t) = \sum_{k=0}^{\infty} P_{k+N,k,B}(N,t)"$,

$$"P_V(N,t) = \sum_{k=0}^{\infty} P_{k+N,k,V}(N,t)\ "$$

3. The "server's utilization time, server's vacation time i.e. the fraction of time the server is busy & the fraction of time server is on vacation until time t can also be expressed in terms of $P_{n,k}(r,t)$"

Server's utilization time is:

$$"U(t) = \sum_{n=0}^{\infty}\sum_{k=0}^{n} P_{n,k,B}(r,t)"$$

Server's vacation time:

$$"V(t) = \sum_{n=0}^{\infty}\sum_{k=0}^{n} P_{n,k,V}(r,t)\ "$$

VII. PERFORMANCE INDICES

(i) The mean number of units in the system is given by

$$"L_s(t) = \sum_{N=1}^{\infty} N\ \ (P_B(N,t) + P_V(N,t))"$$

(ii) The mean number of units in the queue is given by

$$"L_q(t) = \sum_{N=1}^{\infty} (N-1)\ (P_B(N,t) + P_V(N,t))"$$

(iii) The throughput is

$$"T(P) = \sum_{N=1}^{\infty} \mu\ \ (P_B(N,t) + P_V(N,t))"$$

(iv) Mean balking rate is:

$$\text{“}\ B.R. = \sum_{N=1}^{\infty} \lambda(1-\beta)\,(P_B(N,t) + P_V(N,t))\text{”}$$

(vi) Mean reneging rate:

$$"R.R. = \sum_{n=1}^{\infty} \xi(N-1)\ \ (P_B(N,t) + P_V(N,t)"$$

(vii) "Average rate of customer loss is given by:
L.R. = B.R. + R.R."

A. *Cost Model*

We construct an expected cost function for the system.

Let

C1 = server is on vacation.
C2 = server is busy.
C3 = server is idle.
C4 = customer is waiting for service.
C5 = customer joins the system and is served.
C6 = customer balks or renege.

$E(C)= C1*P_{VAC}+C2*P_{BUSY}+C3*P_{IDLE}+C4*E(Lq)+C5*(E(L)-E(Lq))+C6*L.R.$

We fix the maximum number of customers in the system N = 3 and the cost elements "C1 = 100, C2 = 110, C3 = 120, C4 = 150, C5 = 130, C6 = 140". The results for the expected cost C are illustrated below.

VIII. Graphical Presentation

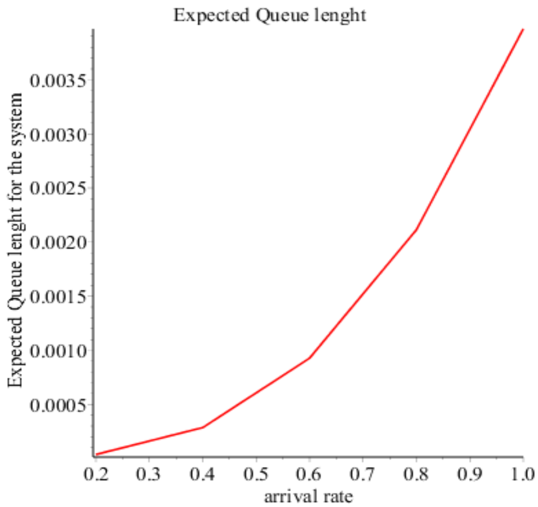

Fig1: Arrival Rate vs. E(L)

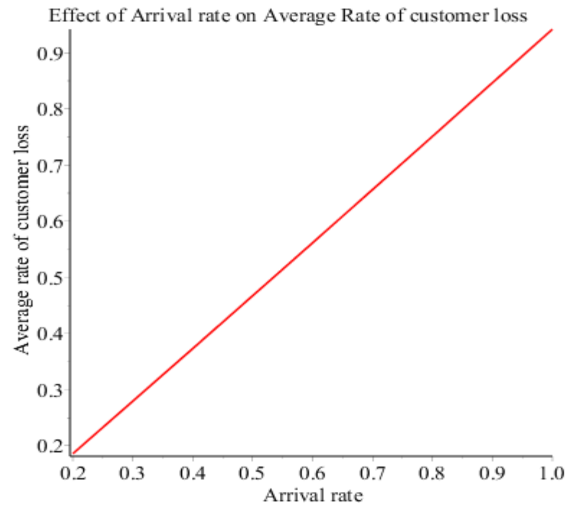

Fig3: Arrival Rate vs. L.R.

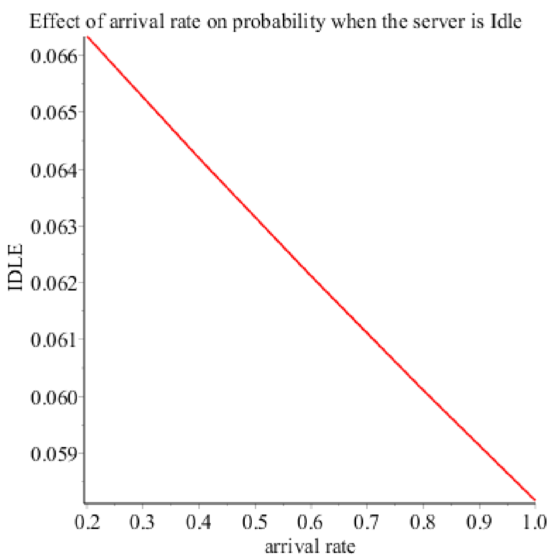

Fig:2 Arrival Rate vs. IDle

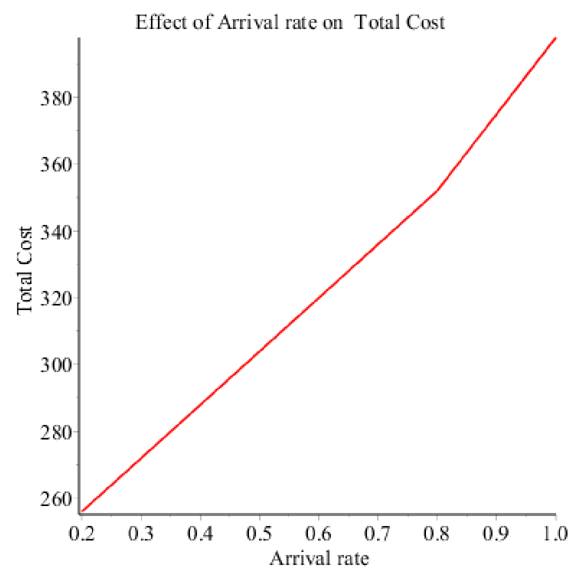

Fig:4 Arrival Rate vs. E(C)

In Fig. 1 to 4 we fix $\mu=1$, $w=0.2$, $i=1$, $\beta=0.2$, $\xi=0.3$, $t=1$ and vary the values of λ. The E(L) in the system, E(C) and L.R. increase as arrival rate increases but Probability of server remains idle decreases as λ increases.

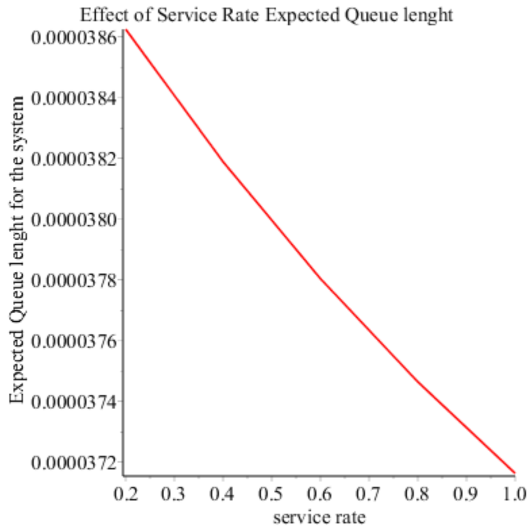

Fig.5: Service Rate vs. E(L)

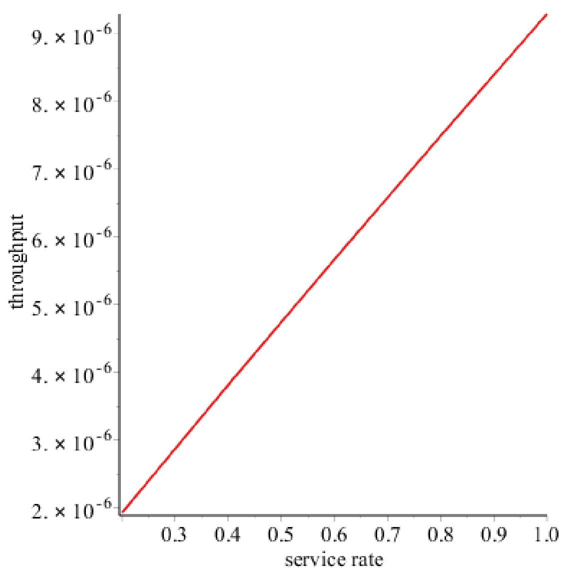

Fig.6: Service Rate vs. T(P)

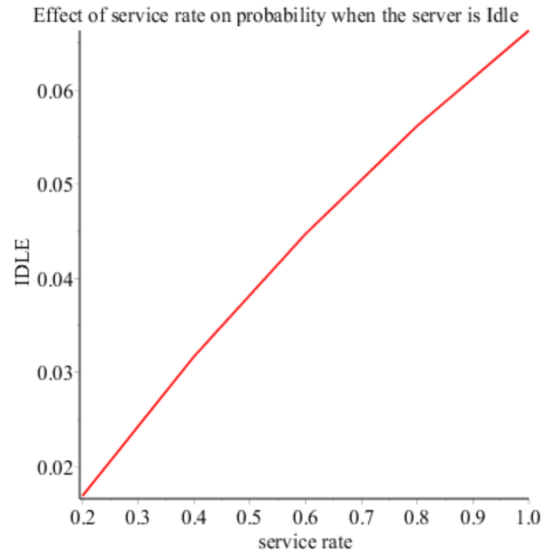

Fig.7: Service Rate vs. Idle

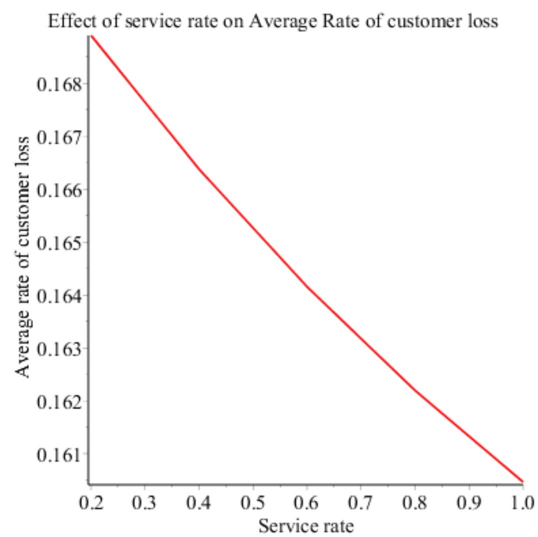

Fig.8: Service Rate vs. L.R.

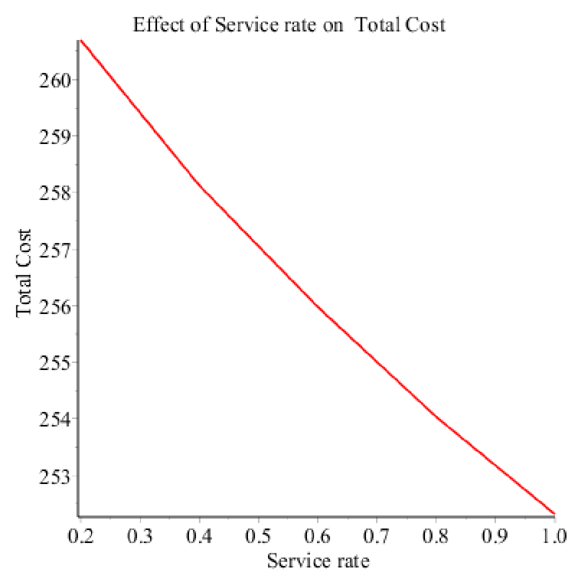

Fig. 9: Service Rate vs. T(C)

In Fig. 5 to 9 we fix λ=0.2, w=0.2, i=1, β=0.2, ξ=0.3, t=1and vary the values of μ. The E(L) in the system, E(C) and L.R. decrease as μ increases but Probability of server remains idle and T(P) increase as μ increases.

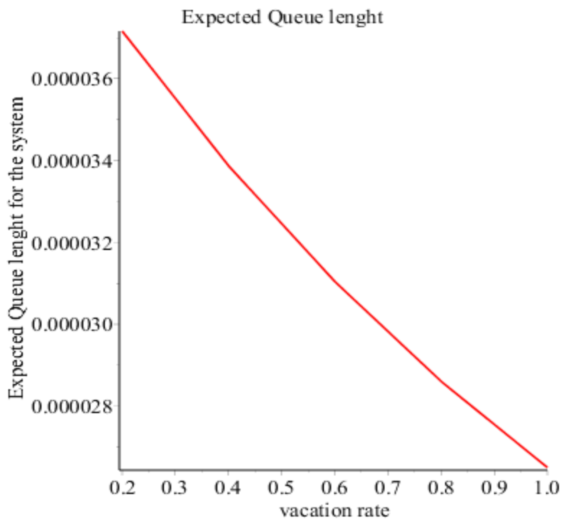

Fig. 10: Vacation Rate vs. E(L)

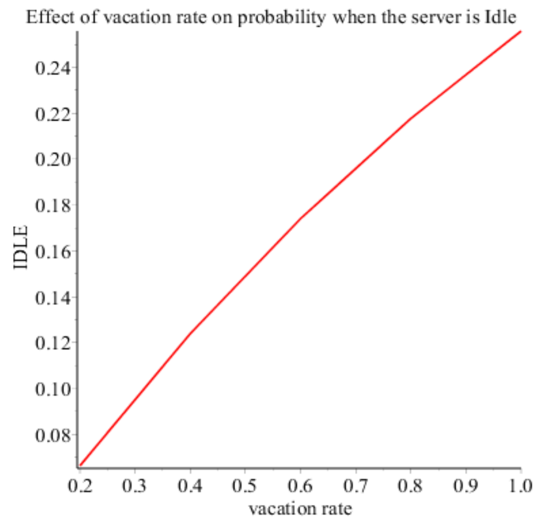

Fig. 11: Service Rate vs. Probability that server remains idle

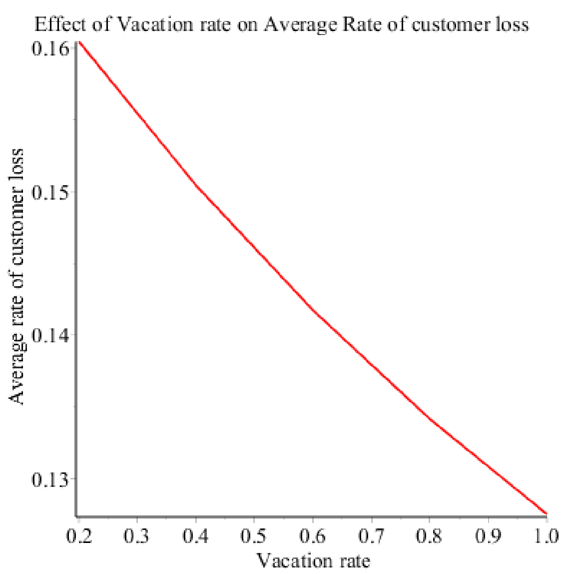

Fig. 12: Vacation Rate vs. L.R.

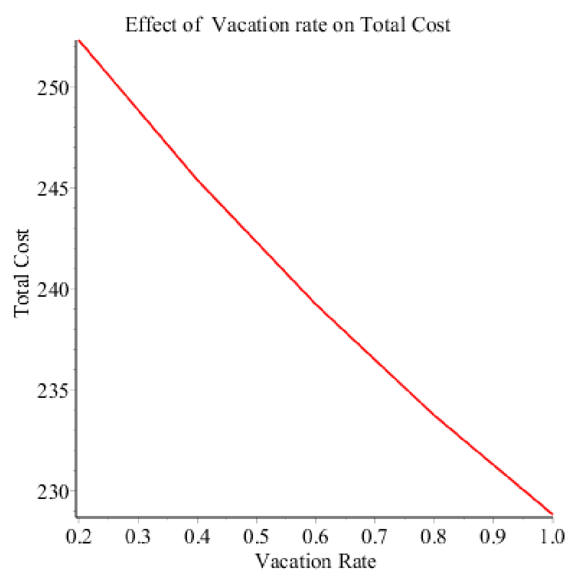

Fig. 13: Vacation Rate vs. E(C)

In Fig. 10 to 13 we fix λ=0.2, μ=1, i=1, β=0.2, ξ=0.3, t=1and vary the values of w. The E (L), E(C) and L.R. decrease as w increases but Probability of server remains idle decreases as w increases.

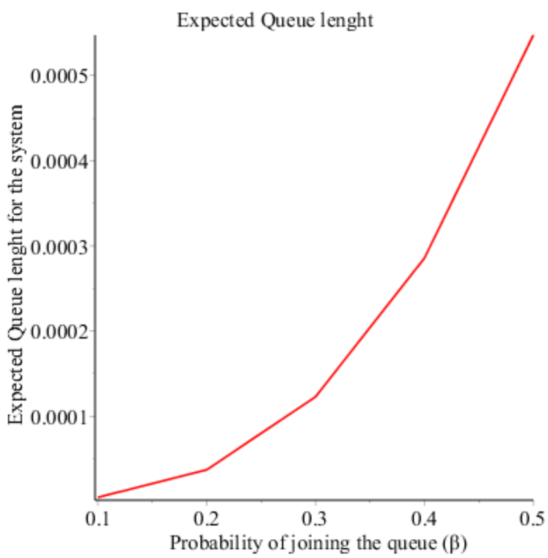

Fig. 14: β vs. E(L)

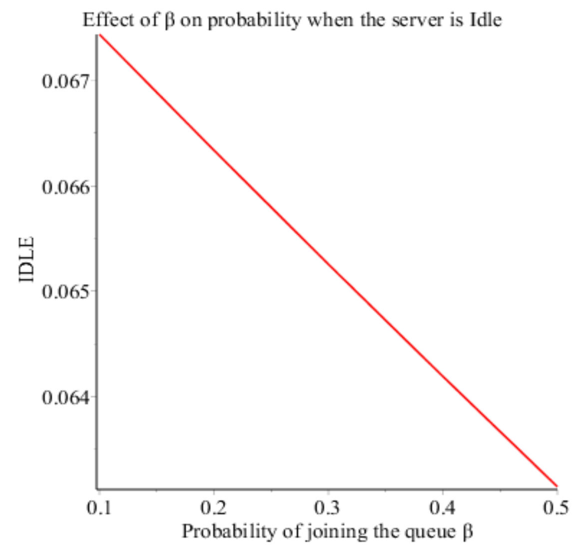

Fig. 15: β vs. Idle

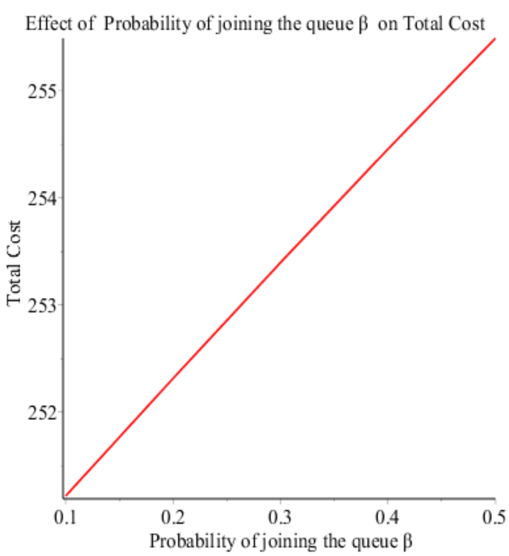

Fig. 16: β vs. E(C)

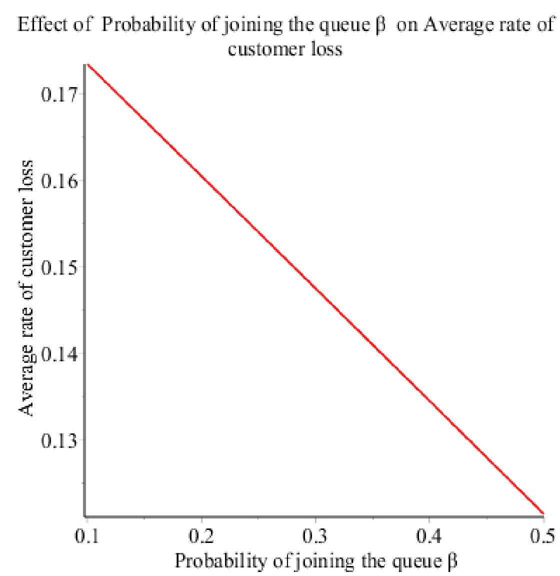

Fig.17: β vs. L.R.

In Fig. 14 to 17 we fix $\lambda=0.2$, $\mu=1$, $w=0.2$, $i=1$, $\xi=0.3$, $t=1$, and vary the values of β. The E(L) and E(C) increase as β increases but Probability of server remains idle and L.R. decrease as β increases.

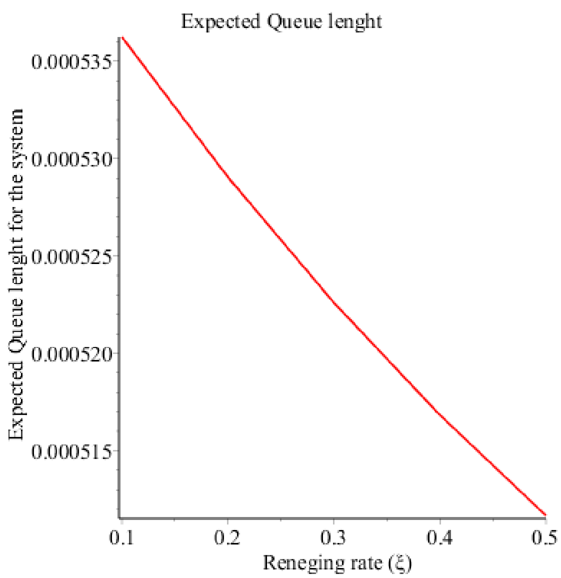

Fig. 18: Reneging Rate vs. E(L)

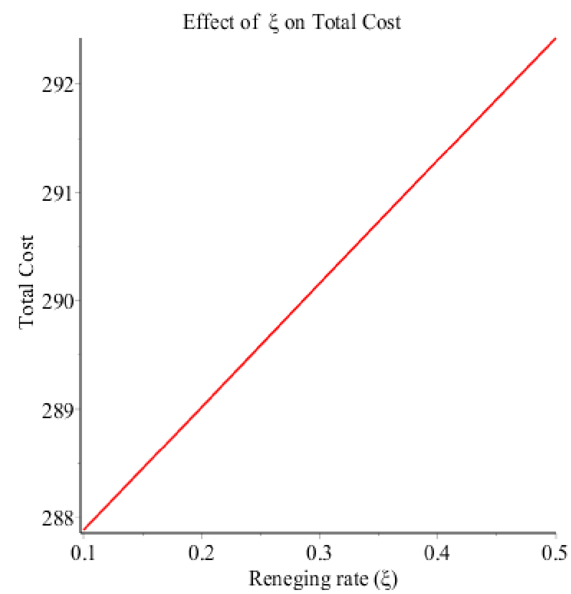

Fig. 19: Reneging Rate vs. E(C)

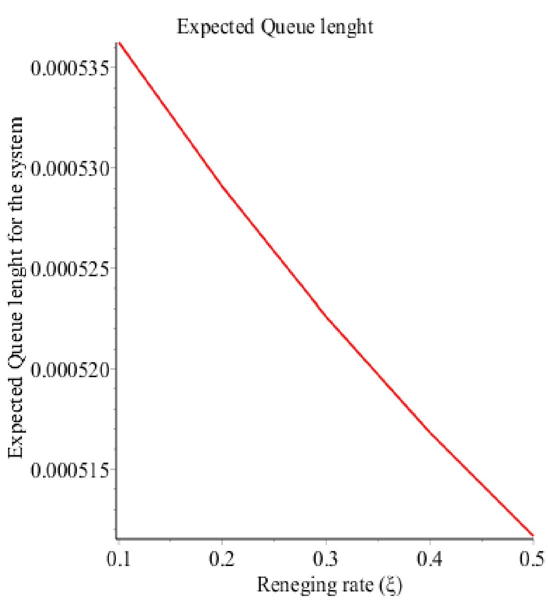

Fig. 20: Reneging Rate vs. E(L)

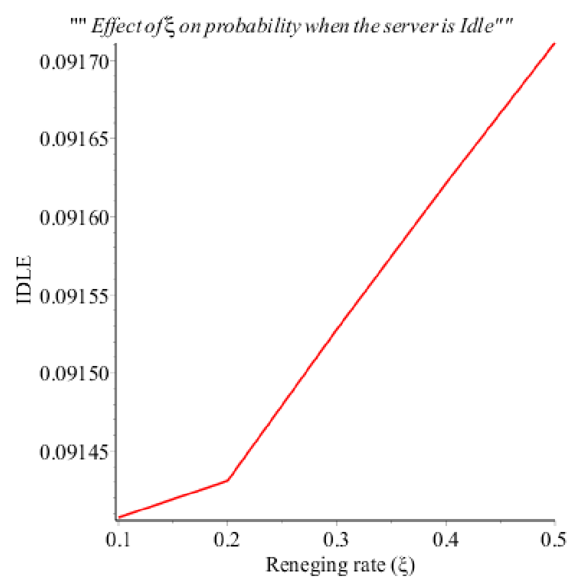

Fig. 21: Reneging Rate vs. Idle

In Fig. 18 to 21 we fix λ=0.2, w=0.2, i=1, β=0.2, t=1, μ=1, and vary the values of ξ. E(C) , probability of server remains idle and L.R. decrease as ξ increases but E(L) decreases as ξ increases.

IX. Conclusion

The present work investigates the "two dimensional state markovian queueing model" with impatient customers and multiple vacations. The assumption "initially few clienteles present in the system" makes this model more applicable in real life congestion problems like telecommunication, post office and health care centers etc. Numerical results have been calculated and presented in tabular forms that displayed the validation of the model with the existing results. Finally graphical presentations display the effect of different parameters on cost function and it reveals that if we increase our service rate the probability of reneging from the system is reduced which results in decrease in expected cost for the system.

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