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Finite-time synchronization between finance hyper-chaotic systems with hyperbolic nonlinearity via adaptive control

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Abstract: This paper examines the finite-time synchronization of identical hyper-chaotic finance systems with hyperbolic nonlinearity. The adaptive control technique has been used to guarantee the synchronization between master and slave systems. The Lyapunov stability theory has been used to stabilize the proposed controller. Some numerical resultsare depicted through graphs, which validates the applicability and efficiency of the suggested technique.

Keywords: Finite time synchronization, adaptive control method, hyper-chaotic finance system.

1. Introduction

Since several decades, nonlinear dynamics has a significant role in the field of mathematics and physics. The dynamical system is being used to describe the evolution of natural phenomena. The predictive power of these sorts of models reflects their utility in scientific and technical applications. One of the essential property of a chaotic system from application point of view is that extremely sensitive to initial conditions. Secure communications can make advantage of dynamical systems that are highly dependent on the initial conditions [1]. It has a wide range of applications in physics, biology, secure communication, engineering, science, sociology, control theory, neural networks and other domains. In 1990, Pecora and Carroll [2] laid the foundation in this unexplored area with their study of synchronization of chaotic systems. A pioneer work was done by Pecora and Carrol [2, 3], to utilize the chaotic systems in communication. The application aspect motivates the researchers to study the chaotic systems. Lorenz discovered the first chaotic attractor in 1963 [4], after that researchers have realized that chaos exists everywhere. In 1999, Chen and Ueta [5] discovered another similar chaotic attractor that was not topologically identical to the Lorenz chaotic attractor but dual to the Lorenz system.

It is well-known fact that there are so many dynamical systems encountered in engineering and physical sciences can be modeled by systems of ordinary nonlinear differential equations. For the engineers and scientists involved in modeling, analysis and design, complex dynamical systems creates big problem. Generally, the dynamical systems represented by nonlinear differential equations are so complex that they cannot be solved analytically in a closed form. Thus, the alternative method of analysis is required in order to ascertain the qualitative behavior of an equilibrium point of a dynamical system. Complex dynamical systems have received more and more attention in recent years, almost all kinds of areas existing in various phenomena in the world, like social network, telecommunication and world wide web (WWW), etc. Since the early 1990s researchers have realized that chaotic systems can be synchronized. The study of synchronization between chaotic systems becomes a herculean endeavour since synchronization can be disrupted by uncertainty and disturbances. The recognized potential for communications systems, this phenomenon has evolved into its own subfield of nonlinear dynamics, with the necessity to comprehend the

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phenomenon in its most fundamental form considered as being essential.

Based on new control techniques, Ouannas and Abu-Saris [6] established certain necessary conditions and got some synchronization criteria. Several scholars have looked into matrix projective synchronization [7–10] and systems with uncertainty and external disturbance [11–14].

Chaos and synchronization have been intensively studied in science and engineering. Chaos control attempts to eradicate chaotic behaviors while synchronization intends to control a slave system to follow master system. A number of control techniques such as adaptive control [15], active control [16-17], back stepping control [18], sliding mode control [19-20], have been used by several researchers.

Sometimes, the parameters of a chaotic system have time-varying property and not always accessible. Over the last decade, adaptive control is one of the most widely used design methods, the adaptive back stepping control method is efficient and convenient to synchronize chaotic systems. Wang and Ge [21] studied synchronization between two uncertain chaotic systems using adaptive back stepping method. They consider master system as smooth, bounded, nonlinear chaotic system, while the slave system in the strict-feedback form. Vaidyanathan et al. [22] global chaos synchronization has been achieved for a pair of new chaotic jerk systems with three nonlinear terms via adaptive back stepping control. Windmi and Coullet chaotic systems has been discussed by Rasappan and Vaidyanathan [23] using adaptive backstepping control method. It is also shown that the adaptive backstepping control method works well and is practical for synchronising and estimating the chaotic systems' parameters. Tirandaz [24] used the adaptive control method to study the synchronization and control of a Zhang chaotic system with uncertain parameters. Huang et al. [25] applied the concept of adaptive control on sliding mode controller and designed master-slave modified Chua's systems. Aghababa and Hashtarkhani [26] studied adaptive control scheme for two distinct chaotic systems with uncertainty and an adaptive control scheme has been designed based on the chaotic systems' state boundedness property. Vaidyanathan [27] investigated the hybrid synchronization of chaotic Liu and Lü systems by adaptive control. An adaptive control technique has also been used by Li and Tong [28] to synchronize a fractional-order chaotic system and find that presented schemes are simple and flexible.

In many scientific domains, hyper-chaos has recently gained attention. In 1985, the chaotic phenomenon was found in economics. In the context of the economic crisis, the chaotic finance system has been investigated. In 2012, Yu et al. [29] added a new state variable to the finance system in order to examine a new, hyper-chaotic financial system and studied equilibrium, stability, Lyapunov exponents, bifurcation analysis of this system. Further, this hyper-chaotic finance system with bounded uncertainties and external disturbances also studied by Cai et al. [30] via chatter free sliding mode control. Jahanshahi et al. [31] investigated the dynamic behaviour of this finance system for several parameters making use of Lyapunov exponents, bifurcation diagrams and phase portraits. Motivated by the above discussions, the authors have studied synchronization between hyper-chaotic systems via adaptive control method.

Finite time synchronization for systems with external disturbance and uncertainty becomes more difficult to achieve than other types of synchronization. The finite time synchronization provides a wide range of applications that improve message security. The finite time synchronization approach is used to secure communication through better chaotic masking. Before being broadcast, the information signal is mixed with the chaotic signal and recovered without distortion by the synchronized receiver.

All types of identical synchronization, in which two or more dynamical systems behave in the same way at the same time, are just examples of dynamical activity restricted to a flat hyper plane in phase space. Whether the behaviour is chaotic, periodic, fixed point, or something else, this holds true.

This paper is organized as follows. The section 2, concerned with the description of the hyper-chaotic finance system. The phase portrait of the hyper-chaotic finance system is depicted through Fig.1. Finite time synchronization of identical hyper-chaotic finance system with hyper-chaotic nonlinearity via adoptive control technique as been studied in Section 3. Section 4 deals with the numerical results and discussion. The conclusion of this research article has been incorporated in section 5.

2. System's description

A dynamic model of finance is system of four first-order differential equations. The hyper-chaotic finance system [29] can be expressed as

$$\dot{\eta}_1(t) = \eta_3 + (\eta_2 - a)\eta_1 + \eta_4,$$
$$\dot{\eta}_2(t) = 1 - b\eta_2 - \eta_1^2,$$
$$\dot{\eta}_3(t) = -\eta_1 - c\eta_3,$$
$$\dot{\eta}_4(t) = -d\eta_1\eta_2 - k\eta_4,$$

The finance hyper-chaotic system with hyperbolic nonlinearity is considered as master system and expressed as

$$\dot{\eta}_1(t) = \eta_3 + (\eta_2 - a)\eta_1 + \eta_4 - \sinh(0.1\eta_1),$$
$$\dot{\eta}_2(t) = 1 - b\eta_2 - \eta_1^2,$$
$$\dot{\eta}_3(t) = -\eta_1 - c\eta_3, \text{ (1) } \dot{\eta}_4(t) = -d\eta_1\eta_2 - k\eta_4,$$

where a, b, c, d and k are positive constants and thefinance hyper-chaotic system with hyperbolic nonlinearity shows chaotic behavior for $a = 0.9$, $b = 0.2$, $c = 1.5$, $d = 0.2$ and $k = 0.17$ .The phase portrait of system (1) is revealed through Fig1.

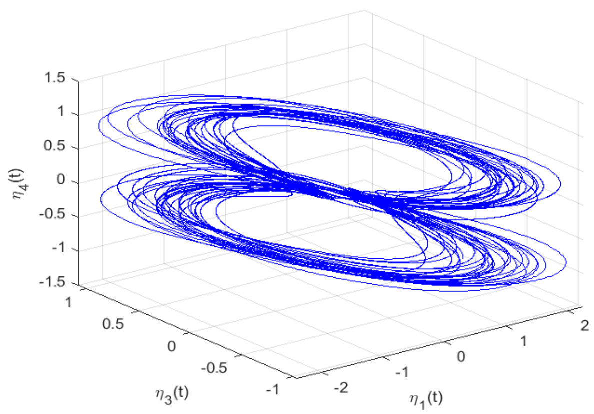


Fig1. Phase portrait of finance hyper-chaotic system in $\eta_1 - \eta_3 - \eta_4$ space.

3. Finite time synchronization between finance systems

This section deals with the study of finite time synchronization between identical hyper-chaotic finance systems with hyperbolic nonlinearity (Fig.1). During the study of finite time synchronization between identical hyper-chaotic finance systems with hyperbolic nonlinearity (1) is taken as master system and the controlled finance system with hyperbolic nonlinearity (2) is taken as slave system

$$\dot{\xi}_1(t) = \xi_3 + (\xi_2 - a)\xi_1 + \xi_4 - \sinh(0.1\xi_1) + u_1(t),$$
$$\dot{\xi}_2(t) = 1 - b\xi_2 - \xi_1^2 + u_2(t), \text{ (2)}$$
$$\dot{\xi}_3(t) = -\xi_1 - c\xi_3 + u_3(t),$$
$$\dot{\xi}_4(t) = -d\xi_1\xi_2 - k\xi_4 + u_4(t),$$

Define the error system as $e_i = \xi_i - \eta_i$, $\forall i = 1,2,3,4$ .The error system is defined as

$$\dot{e}_1(t) = -ae_1 + e_3 + e_4 + \xi_1\xi_2 - \eta_1\eta_2 - \sinh(0.1\xi_1) + \sinh(0.1\eta_1) + u_1(t),$$
$$\dot{e}_2(t) = -be_2 - \xi_1^2 + \eta_1^2 + u_2(t), \text{ (3)}$$
$$\dot{e}_3(t) = -e_1 - ce_3 + u_3(t),$$
$$\dot{e}_4(t) = -ke_4 - d\xi_1\xi_2 + d\eta_1\eta_2 + u_4(t).$$

Now define the adaptive control function which are obtained from Eq. (3), given as

$$u_1(t) = \hat{a}e_1 - e_3 - e_4 - \xi_1\xi_2 + \eta_1\eta_2 - \sinh(0.1\eta_1) - k_1e_1 - m_1 \operatorname{sgn}(\xi_1 - \eta_1)|\xi_1 - \eta_1|^{\gamma},$$

$$u_2(t) = \hat{b}e_2 + \xi_1^2 - \eta_1^2 - k_2e_2 - m_2 \operatorname{sgn}(\xi_2 - \eta_2)|\xi_2 - \eta_2|^{\gamma},$$
(4) $$u_3(t) = e_1 + \hat{c}e_3 - k_3e_3 - m_2 \operatorname{sgn}(\xi_3 - \eta_3)|\xi_3 - \eta_3|^{\gamma},$$
$$u_4(t) = \hat{k}e_4 + \hat{d}(\xi_1\xi_2 - \eta_1\eta_2) - k_4e_4 - m_4 \operatorname{sgn}(\xi_4 - \eta_4)|\xi_4 - \eta_4|^{\gamma},$$

where $\hat{a}, \hat{b}, \hat{c}, \hat{d}$ are estimation of parameters a, b, c, d respectively and k_1, k_2, k_3, k_4 are positive constants.

$$\dot{e}_1(t) = -(a - \hat{a})e_1 - k_1e_1,$$
$$\dot{e}_2(t) = -(b - \hat{b})e_2 - k_2e_2, \text{ (5) } \dot{e}_3(t) = -(c - \hat{c})e_3 - k_3e_3,$$
$$\dot{e}_4(t) = -(k - \hat{k})e_4 - (d - \hat{d})(\xi_1\xi_2 - \eta_1\eta_2) - k_4e_4.$$

Parameters estimation errors are given as

$$e_a = a - \hat{a}, e_b = b - \hat{b}, e_c = c - \hat{c}, e_d = d - \hat{d}, e_k = k - \hat{k}$$
.(6)

The derivatives of these function are expressed as

$$\dot{e}_a = -\dot{\hat{a}}, \dot{e}_b = -\dot{\hat{b}}, \dot{e}_c = -\dot{\hat{c}}, \dot{e}_d = -\dot{\hat{d}}, \dot{e}_k = -\dot{\hat{k}} \text{ .(7)}$$

Putting the values of $a - \hat{a}, b - \hat{b}, c - \hat{c}, d - \hat{d}, k - \hat{k}$ from Eq. (6) to Eq. (5), we get

$$\dot{e}_1(t) = -e_ae_1 - k_1e_1,$$
$$\dot{e}_2(t) = -e_be_2 - k_2e_2, \text{ (8)}$$
$$\dot{e}_3(t) = -e_ce_3 - k_3e_3,$$
$$\dot{e}_4(t) = -e_ke_4 - e_d(\xi_1\xi_2 - \eta_1\eta_2) - k_4e_4.$$

Now we calculate the Lyapunov function, which is given as

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2 + e_k^2)$$
(9)

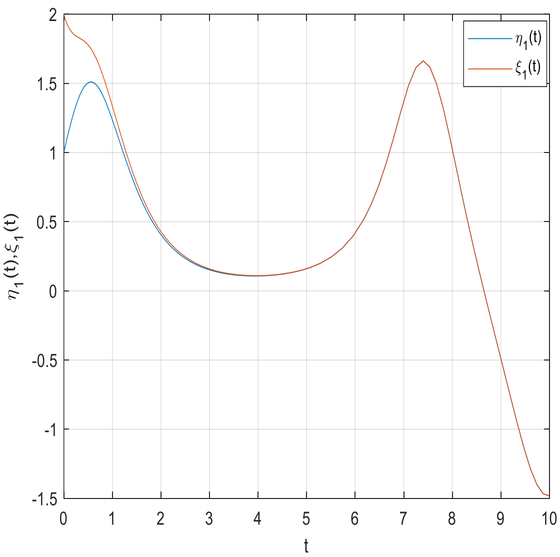
The derivative of Lyapunov function is

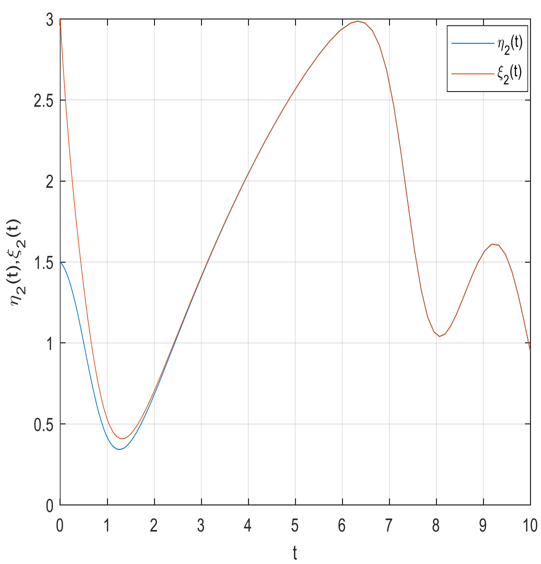
$$\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 - e_a(e_1^2 + \dot{\hat{a}}) - e_b(e_2^2 + \dot{\hat{b}}) - e_c(e_3^2 + \dot{\hat{c}}) - e_d(e_4(\xi_1\xi_2 - \eta_1\eta_2) + \dot{\hat{d}}) - e_k(e_4^2 + \dot{\hat{k}}).$$
(10)

Now the estimated parameters $\hat{a}, \hat{b}, \hat{c}, \hat{d}$ and $\hat{k}$ can be obtained from Eq. (10)

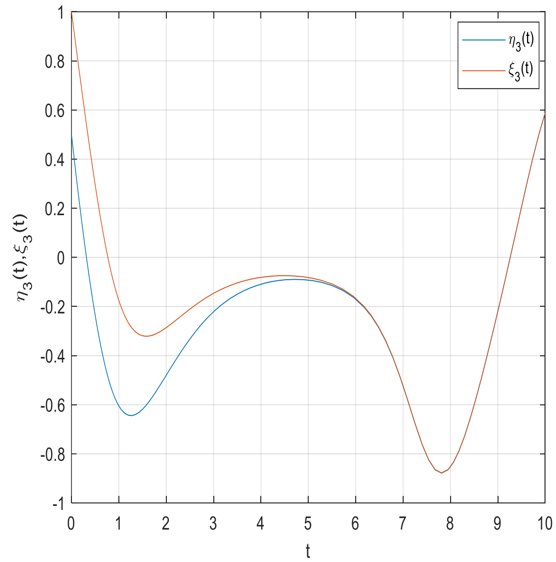
$$\begin{aligned}
\dot{\hat{a}} &= -e_1^2 + k_5 e_a, \\
\dot{\hat{b}} &= -e_2^2 + k_6 e_b, \\
\dot{\hat{c}} &= -e_3^2 + k_7 e_c, \qquad (11) \\
\dot{\hat{d}} &= -e_4(\xi_1\xi_2 - \eta_1\eta_2) + k_8 e_d, \\
\dot{\hat{k}} &= -e_4^2 + k_9 e_k,
\end{aligned}$$

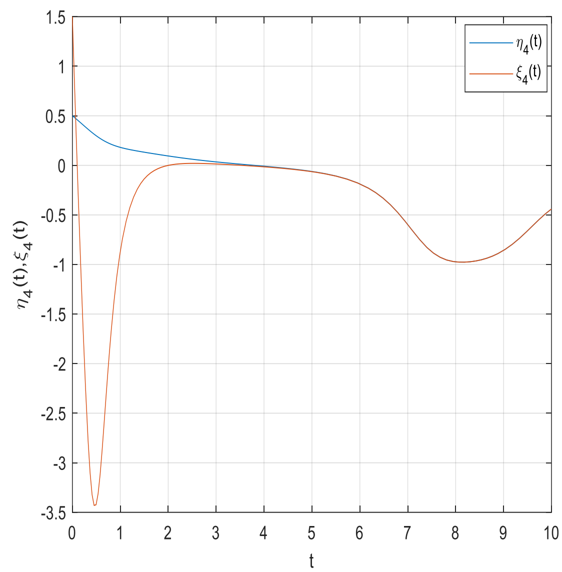
where k_5, k_6, k_7, k_8, k_9 are positive constants.



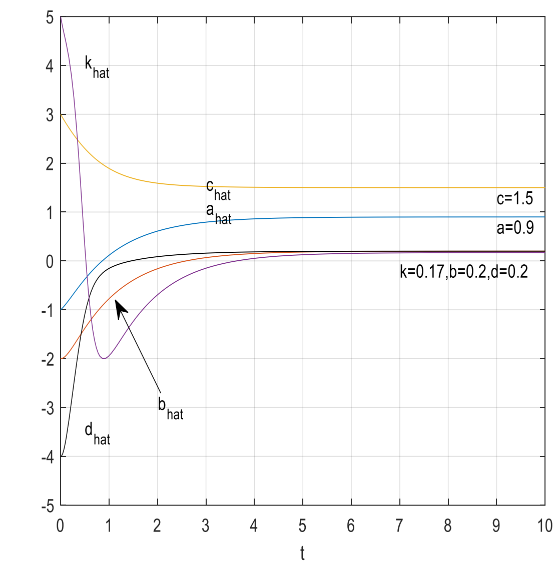


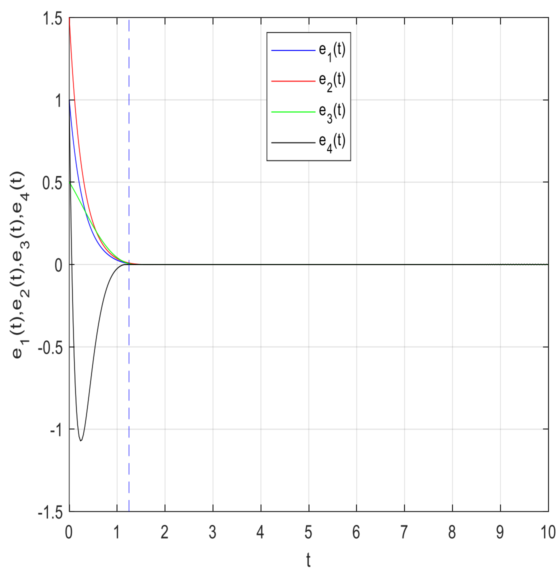
(a)
(b)





(c)
(d)





(e) (f)

Fig. 2.synchronization (a) between $\eta_1(t)$ and $\xi_1(t)$, (b) between $\eta_2(t)$ and $\xi_2(t)$, (c) between $\eta_3(t)$ and $\xi_3(t)$, (d) between $\eta_4(t)$ and $\xi_4(t)$,(e) Parameter estimation of $\hat{a},\hat{b},\hat{c},\hat{d}$ and $\hat{k}$ (f) The time evolution of state errors $e_1(t), e_2(t), e_3(t), e_4(t)$.

Now the derivative of Lyapunov function is obtained as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 - k_5 e_a^2 - k_6 e_b^2 - k_7 e_c^2 - k_8 e_d^2 - k_9 e_k^2. \quad (12)$$

Thus, the error dynamics (5) are globally exponentially stable, according to Lyapunov stability theory [19].

4. Numerical results and discussion

In this work, all the numerical simulations have been done for solving the system using MATLAB ode45 with time step 0.0005. In adaptive synchronization, the parameter's values of the finance system are taken as $a = 0.9, b = 0.2$, $c = 1.5, d = 0.2$ and $k = 0.17$ respectively.

The initial values of hyper-chaotic finance system are taken as $(\eta_1(0), \eta_2(0), \eta_3(0), \eta_4(0)) = (1, 1.5, 0.5, 0.5)$, and $(\xi_1(0), \xi_2(0), \xi_3(0), \xi_4(0)) = (2, 3, 1, 1.5)$.

The value of the constants used in adaptive control function are taken as $k_1 = 1,\ k_2 = 1,\ k_3 = 1,\ k_4 = 8,\ k_5 = 1,\ k_6 = 1,\ k_7 = 1,\ k_8 = 1,\ k_9 = 1; m_1 = 0.5,\ m_2 = 0.5,\ m_3 = 1,$

$m_4 = 0.5;\ \gamma = 0.5;$

The phase portraits of hyper-chaotic finance system is demonstrated by Fig.1 and the Figs.(2a- 2d) depicts the synchronization between states of signals. The graphical presentation of estimation of parameters $\hat{a},\hat{b},\hat{c},\hat{d}$ and $\hat{k}$ has been depicted through Fig. 2(e). The errors with respect to time between identical hyper-chaotic finance systems graphically represented by Fig. 2(f),which shows that error states converges to zero after a small time duration. This property can be used in communication technology i.e. the fast communication can be achieved between transmitter and receiver signals. Therefore, in proposed hyper-chaotic finance system with hyperbolic uncertainties the communication between transmitter and receiver will be very fast.

5. Conclusion

The present manuscript has successfully demonstrated the finite time synchronization between identical hyper-chaotic finance systems with hyper-chaotic nonlinearity through adoptive control method. The control function is designed in such a way that the error states tend to zero when time tends to infinity. The graphs (Figs.2a-2d) demonstrate the finite time synchronization between master and slave systems, which validates the theoretical results with computational results. The graphical presentation of numerical results clearly reveals that the applied method is much more reliable and convenient to achieve synchronization. The authors' are optimist that this work will be quite beneficial for the researchers working in field of dynamical systems.

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