Volume 66, Issue 4, 2022
Journal of Scientific Research
of
The Banaras Hindu University

Dynamic DEA Model using Window Analysis Approach with Weight Modeling

Seema Gupta

Bhagini Nivedita College, University of Delhi. seemadavv@gmail.com

Abstract: **This paper study dynamic DEA model and proposes weight modeling in window analysis for multi-time period performance evaluation. It uses the weights of inputs and outputs generated as a result of static temporal efficiency evaluation to restrict weight flexibility in dynamic DEA model and observe the efficiencies, thus obtained, over multi-time period. First the optimal range of weights are determined and then it is being applied in different windows to observe the stability and consistency of efficiency evaluations. We also define consistent efficiency index and give numerical illustration to observe the working of proposed model.**

Index Terms: **Dynamic DEA, dimensional space, Window Analysis, weight modeling, Efficiency index.**

I. Introduction

Window analysis approach offers an opportunity to quantify the efficiency score with respect to own performance over different sequence of overlapping time period as well as the performance of the others (Flokou et al.,2017). Original model of DEA as defined by Charnes, Cooper and Rhodes in 1978 for efficiency evaluation using weighted sum of multiple outputs to weighted sum of multiple inputs is static in nature (Charnes et al.,1978). Decision Making Units (DMs) is a data point in $m + s$ dimensional space, wherein m are the number of inputs and s are number of outputs (Charnes et al.,1981). However, there are certain evaluation scenarios, where the decision maker is interested in finding the efficiency evaluation and observed thee DMU over multiple time period. For example, in a bank branches performance evaluation, the general manager not only want the branches performance over a single time data point but more interested in finding long-term efficiency and thereby allocate or reallocate the future resources accordingly. Thus, static DEA will not work in this case. Therefore, Dynamic DEA needs to be applied in such scenario. This measurement of temporal efficiency evaluation is offered by several techniques like window analysis approach introduced and developed by (Banker et al.,1984; Charnes et al. 1984; Klopp, 1985), Malmquist index (Malmquist, 1953) in DEA framework (Chang et al., 2009), slake based measure for dynamic DEA (Pastor et al.,1999) and (Tone, 2003), Network DEA (F¨are, 1996; Kao, 2008) to mention few. Various other approaches are found in (Nemoto & Goto, 2003; Sueyoshi & Sekitani, 2005; Bogetoft et al., 2008; Chen, 2009; Park & Park 2009). Rest of the paper is arranged as follows: next section talks about window analysis approach in DEA. After that weight modeling is introduced. Important key terms are stated in next section. Proposed methodological framework along with an algorithm is mentioned in next section. Numerical illustration is provided next with results and discussion. Last section concludes the research work.

II. Window Analysis Approach

Window analysis approach, introduced by (Charnes et al. 1984) and offers an opportunity to quantify the efficiency score with respect to own performance over different sequence of overlapping time period as well as the performance of the others for benchmarking. Since then, various researchers have approached the window analysis methods (Chang et al., 2009; Halkos & Tzeremes, 2009; Kazley & Ozcan, 2009) Window analysis is one of the methods used to verify productivity change over time. Window analysis approach, introduced by Charnes et al. in 1985 offers an opportunity to quantify the efficiency score with respect to own performance over different sequence of overlapping time period as well as the performance of the others for benchmarking. It works on the principle of moving averages (Yue, 1992; Charnes et al., 1997; Cooper et al., 2007). It is useful in detecting performance trends of a decision-making unit over time. Each DMU is treated as a different entity in a different period which increases the number of data point. i.e. each DMU in a different period is treated as if it were a independent DMU but remain comparable in the same window (Cooper et al., 2011). Hence in case of a small number of DMUs and a large number of inputs and outputs, this technique also satisfy the rule of thumb in DEA and increases the discriminatory power of the DEA models (Cooper et al., 2011).

III. Weight Modeling

Weight modeling can be achieved by weight restrictions via Absolute weight restriction, Assurance region of Type I, Assurance region of type II and virtual weight restrictions. (Dyson & Thanassoulis, 1988) applied absolute weight restrictions in DEA analysis to reduce the weight flexibility. To evaluate highway maintenance patrols, Cook et al., 1994; also used this type of constraints. One can refer (Podinovski, 2016) for various aspects on weight modeling. Weight modeling is applied in DEA to include preferences in the decision making. Sometime these preferences are specified on prior basis. Sometime introduced as measure to reduce flexibility. For review of such weight modeling can be refereed in (Angulo-Meza & Lins, 2002) also. The advantage of introducing weight modeling is to provide greater discriminatory power to DEA.

IV. Important Definitions and Key Terms

A. *Input and output of j^{th} DMU at time t:*

Consider N DMUs (n=1,2,…,N) observed over T time periods(t=1,2,….,T). Each DMU in each time period has m outputs and s inputs. So, a DMU j at t time period has input vector and output vector as

$$X_j^t = (x_{1j}^t, x_{2j}^t, \dots, x_{sj}^t) \qquad (1)$$

$$Y_j^t = (y_{1j}^t, y_{2j}^t, \dots, y_{mj}^t)$$

B. *Window starting from kth time period with width w:*

A window is defined as collection of time observations for more than one time period. Thus, a window k_w with $k \times w$ time period within the observed time periods T is defined as follows:

If k is a starting time point i.e. $1 \le k \le T$ and w is the width of window i.e. $1 \le w \le T - k$.
Then window $k_w = (k, k+1, k+2, \dots \dots, k+w)$. (2)

C. *Input Matrix for N DMUs in window k_w:*

If input matrix at time t is of order $N \times m$ then then input matrix in k_w window will be of order $(N+w) \times m$

$$X_{kw} = (x_k^1, x_k^2, \dots, x_k^N, x_{k+1}^1, x_{k+1}^2, \dots, x_{k+1}^N, \dots, x_{k+w}^1, x_{k+w}^2, \dots, x_{k+w}^N) \qquad (3)$$

Here each x_{k+i}^n represent the input column for n[th] DMU at k+i time period, i=1,2,…,w.

D. *Output Matrix for N DMUs in window k_w:*

A If output matrix at time t is of order $N \times m$ then then output matrix in k_w window will be of order $(N+w) \times m$

$$Y_{kw} = (y_k^1, y_k^2, \dots, y_k^N, y_{k+1}^1, y_{k+1}^2, \dots, y_{k+1}^N, \dots, y_{k+w}^1, y_{k+w}^2, \dots, y_{k+w}^N) \qquad (4)$$

Here each y_{k+i}^n represent the output column for n[th] DMU at k+i time period, i=1,2,…,w.

Note that in window analysis one DMU at two time period is treated as two separate entities.

E. *Absolute weight restrictions:*

It means that weights are restricted to be within a specific range. i.e. $a_r \le u_r \le b_r$ and $c_i \le v_i \le d_i$ for some constants a_r, b_r, c_i, d_i.

F. *Assurance region of type I:*

It means that relative weights of inputs and outputs are restricted to be within a specific range. i.e. $\alpha_r \le \frac{u_r}{u_{(r+1)}} \le \beta_r$ and $\gamma_i \le \frac{v_i}{v_{(i+1)}} \le \delta_i$ for some constants $\alpha_r, \beta_r, \delta_i, \delta_i$.

G. *Assurance region of type II:*

It means that weights are restricted to be within a specific range. i.e. $\tau_r v_i \le u_r \le b_r$ for some constants τ_r.

H. *Cone Ratio:*

when the weights are restricted to be in polyhedral convex cone. i.e $u_r \in U$ and $v_i \in V$ for some polyhedral cone U and V.

I. *Consistently efficient DMU:*

A DMU is called consistently efficient, if it is efficient in all the time periods and all the different windows, i.e. such DMU has efficiency score always 1 for all time periods t= 1, 2,…., T and all windows k_w.

J. *Consistently inefficient DMU:*

A DMU is called consistently inefficient, if it is inefficient in all the time periods and all the different windows, i.e. such DMU has efficiency score always less than 1 for all time periods t= 1, 2,…., T and all windows k_w.

K. *Modified Dynamic efficiency index:*

The efficiency index as calculated using proposed modified Dynamic DEA model.

V. Proposed Methodology and Algorithm

Proposed methodology includes a new model developed to include weight modeling into window analysis in Dynamic DEA environment. For this, optimal weight bounds are determined using the weight calculated from DEA model run for each time period, taking all DMUs, and for each DMU in all time period. We assume that there are N DMU's (n=1,2,…,N) observed over T time periods(t=1,2,….,T). Each

DMU in each time period has m outputs and s inputs. So input and output vector for DMU n, at time t can be written as

$X_n^t = \begin{bmatrix} x_{1n}^t \\ \vdots \\ x_{mn}^t \end{bmatrix}$ is input vector in $\mathbb{R}^m$, $Y_n^t = \begin{bmatrix} y_{1n}^t \\ \vdots \\ y_{sn}^t \end{bmatrix}$ is a output vector in $\mathbb{R}^s$.

Following algorithm describe the steps for finding weight ranges and mathematical formulation of developed model along with its solution.

A. Algorithm:

Step 1: In each time period, taking all DMUs, we calculate the efficiency score for comparison with other DMUs in each time period. This gives us static efficiency scores to observe the performances of DMUs in a fixed time period.

Static DEA model for time t : efficiency of DMU k into consideration at time t.

$$\theta_{t\_k} = max \sum_{r=1}^{s} u_{rk}^t y_{rk}^t$$

$$\text{s.t.} \quad \sum_{i=1}^{m} v_{ik}^t x_{ik}^t = 1 \qquad (5)$$

$$\sum_{r=1}^{s} u_{rk}^t y_{rn}^t - \sum_{i=1}^{m} v_{ik}^t x_{in}^t \le 0 \qquad for\ n = 1,2,..,N$$

$$u_{rk}^t \ge 0 \qquad for\ r = 1,2,..,s$$

$$v_{ik}^t \ge 0 \qquad for\ i = 1,2,..,m$$

Here u_{rk}^t and v_{ik}^t are the weights corresponding to outputs and inputs as described in (1) for DMU k and θ_{t_k} represents the static efficiency score of DMU k at fixed time t. This static model (5) is run $N \times T$ times for each DMU in T time period.

Step 2: Next temporal efficiency scores are calculated by considering each DMU individually over all the time period (comparison with own over different time periods). Note that the DMU *n* in time period *t* and DMU *n* in time period *(t+1)* are treated as two different units.

Static temporal DEA model for DMU k for all time period: efficiency of DMU k for all time period.

$$\theta_{k\_t} = max \sum_{r=1}^{s} u_{rk}^t y_{rk}^t$$

$$\text{s.t.} \quad \sum_{i=1}^{m} x_{ik}^t x_{ik}^t = 1 \qquad (6)$$

$$\sum_{r=1}^{s} u_{rk}^t y_{rk}^t - \sum_{i=1}^{m} v_{ik}^t x_{ik}^t \le 0 \qquad for\ t = 1,2,..,T$$

$$u_{rk}^t \ge 0 \qquad for\ r = 1,2,..,s$$

$$v_{ik}^t \ge 0 \qquad for\ i = 1,2,..,m$$

Here u_{rk}^t and v_{ik}^t are the weights corresponding to DMU k and θ_{k_t} represents the efficiency measurement at fixed DMU *k*. The model (6) is run $T \times N$ times for each time period once for each DMU.

Step 3: Next weight matrix is formed for each DMU obtained from the above *T+N* models. From the weight matrices, multiplier restrictions corresponding to outputs and inputs for each DMU is determined using (7), (8), (9) and (10).

$$u_{rj\_min} = \min_{T}\{\ u_{rj}^t, t = 1,2,\dots,T\} \quad for\ j = 1,2,\dots,N \qquad (7)$$

$$u_{rj\_max} = \max_{T}\{\ u_{rk}^t, t = 1,2,\dots,T\} \quad for\ j = 1,2,\dots,N \qquad (8)$$

$$v_{ij\_min} = \min_{T}\{\ v_{ik}^t, t = 1,2,\dots,T\} \quad for\ j = 1,2,\dots,N \qquad (9)$$

$$v_{ij\_max} = \max_{T}\{\ v_{ik}^t, t = 1,2,\dots,T\} \quad for\ j = 1,2,\dots,N \qquad (10)$$

At the end of this step, we get output and input weight ranges for all DMUs $j = 1,2,\dots,N$ as given in (11) and (12).

$$\left(u_{rj\_min}\ ,\ u_{rj\_max}\right) \qquad \text{for } r = 1,2,\dots,s \qquad (11)$$

$$\left(v_{ij\_min}\ ,\ v_{ij\_max}\right) \qquad \text{for } i = 1,2,\dots,m \qquad (12)$$

Step 4: The weight ranges obtained in step 3 are used for weight modeling in the window analysis for each DMU in different windows. The windows can be chosen as per the given time series and preferences of Decision Makers. So, for a chosen window of say width *w* starting from time period *k*, as described in equation (2) the following modified dynamic DEA model is proposed by using Window analysis approach with weight modeling:

Modified Dynamic DEA model: efficiency of DMU j in window k_w

$$\theta_{jkw} = max \sum_{r=1}^{s} u_{rj}^{k+w} y_{rj}^{k+w}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} v_{ij}^{k+w} x_{ij}^{k+w} = 1 \qquad (13)$$

$$\sum_{r=1}^{s} u_{rj}^{k+w} y_{rn}^{k+w} - \sum_{i=1}^{m} v_{ij}^{k+w} x_{in}^{k+w} \le 0$$

$$for\ n = 1,2,\dots,N\ and\ w = 0,1,2,..,w$$

$$u_{rj\_min} \le u_{rj}^{k+w} \le u_{rj\_max} \qquad for\ r = 1,2,..,s\ and\ w = 0,1,2,..,w$$

$$v_{ij\_min} \le v_{ij}^{k+w} \le v_{ij\_max} \qquad for\ i = 1,2,..,m\ and\ w = 0,1,2,..,w$$

Using the model (13), the efficiency scores in each window are calculated and tabulated to observe the consistency and stability in performance over time. If a DMU is efficient in all windows, then it is termed as consistently Efficient DMU. i.e. for a consistent efficient DMU p, optimal efficiency score $\theta_{pkw}^* = 1, \forall\ k\ and\ \forall\ w.$

If a DMU is consistently inefficient over all windows, it is termed as consistently inefficient DMU. i.e. for an consistent inefficient DMU g,

$\theta_{gkw}^* < 1, \forall\ k\ and\ \forall\ w$.

For the rest DMUs, consistency and stability of performances can be observed from the efficiency table obtained from modified dynamic model (13).

VI. NUMERICAL ILLUSTRATION

To illustrate the applicability of the proposed algorithm and methodology, we now consider one numerical example with 9 DMUs , having five inputs, two outputs for five time period from 2005 to 2009. The data for the study is compiled form (Sueyoshi & Goto, 2012). The source of data is Handbook of Electric Power Industry (2010).

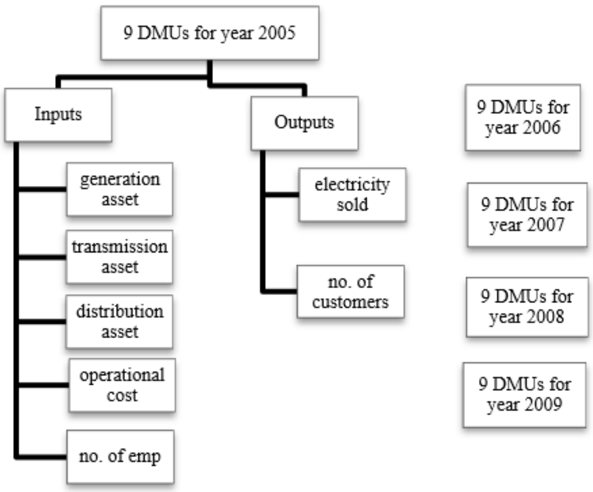

Figure 1 DMUs with their attributes for study

As shown in figure 1, there are 9 Electric Power industries with five inputs as total generation asset(I1), total transmission asset(I2), total distribution asset(I3), Operational cost other than labour cost(I4), total number of employees(I5). Outputs are total electricity sold (O1) and total number of customers(O2). The numerical values of each of the inputs and outputs for 9 DMUs over 5 time period is given in table 1.

Table 1: Input and Output data set

	DMU	I1	I2	I3	I4	I5	O1	O2
2005	DMU1	350336	282900	259817	391268	5844	30833	3870
	DMU2	1070866	1019688	635913	1306421	12263	79664	7642
	DMU3	3001875	3610373	2330292	4180478	38039	288655	27772
	DMU4	1470460	1640015	816895	1676284	16180	130561	10299
	DMU5	674101	323849	156514	392695	4692	27966	1996
	DMU6	1352435	1886939	995636	1950765	22229	147108	13160
	DMU7	587216	685432	435117	797625	10690	59501	5183
	DMU8	350619	346144	216116	426765	6043	27968	2843
	DMU9	838154	1006616	632016	1041298	13066	82956	8286
2006	DMU1	329847	269647	261956	405473	5794	31512	3899
	DMU2	1019688	932300	638177	1351343	12148	80950	7665
	DMU3	2778054	3477710	2314569	4226905	38111	287622	28067
	DMU4	1346481	1572398	809402	1826439	15973	132687	10388
	DMU5	608727	31298	156466	396704	4638	28200	2082
	DMU6	1352435	1791119	976781	2026497	22164	147257	13282
	DMU7	587216	643536	430718	835502	10445	61259	5206
	DMU8	350619	327342	215695	442407	6045	28161	2847
	DMU9	838154	996452	629785	1088982	12660	84399	8349
2007	DMU1	317164	272918	263321	456049	5724	32445	3919
	DMU2	922896	929725	664298	1449628	12155	84072	7665
	DMU3	2587475	3330119	2293329	4950100	38238	297397	28316
	DMU4	1235321	1491268	803103	1970206	15952	137484	10443
	DMU5	552088	298459	160861	416553	4611	29305	2082
	DMU6	1185759	1704998	961181	2180008	22111	150422	13337
	DMU7	541448	617098	435185	892766	10165	63579	5191
	DMU8	298953	309387	215600	462911	6030	29269	2835
	DMU9	867918	959739	629769	1203091	12459	88082	8380
2008	DMU1	310754	272807	266338	558382	5699	31839	3938
	DMU2	862151	908387	660784	1585495	12410	81101	7675
	DMU3	2522816	3181070	2267197	5290057	37913	288956	28491
	DMU4	1185652	1418567	839473	2073085	16221	129734	10459
	DMU5	511717	297649	158909	458655	4630	28154	2081
	DMU6	1104820	1637292	945409	2410097	22106	145867	13396
	DMU7	504351	588712	426027	1025018	9938	61222	5194
	DMU8	273733	293966	215182	473336	6014	28701	2831
	DMU9	857191	931815	630378	1261759	12456	85883	8397
2009	DMU1	543941	267055	269711	459634	5631	31451	3957
	DMU2	800405	883380	664245	1307233	12639	78992	7688
	DMU3	2420191	3044294	2231586	4212776	38117	280167	28599
	DMU4	1095275	1354584	831446	1707687	16600	122849	10455
	DMU5	468902	286755	157363	387334	4716	27175	2084
	DMU6	1119070	1589075	933788	1990398	22143	141605	13432
	DMU7	487284	561094	416440	826141	9871	57911	5197
	DMU8	283326	282070	215744	402193	6003	27496	2833
	DMU9	804843	917957	629742	1124143	12543	83392	8437

As per the proposed algorithm, according to step 1, first static DEA models are formulated and solved for each DMU fixing the time period to observe the performances of the DMUs. Table 2 shows the static efficiency values as calculated from running static model (5). Total $9 \times 5 = 45$ models are solved in this step.

Table 2: Static efficiency index for DMUs in 5 time period

DMUs	Static Efficiency Index				
	2005	2006	2007	2008	2009
DMU1	1	1	1	1	1
DMU2	0.925196	0.9307	0.928136	0.918949	0.926998
DMU3	1	1	1	1	1
DMU4	1	1	1	1	1
DMU5	1	1	1	1	1
DMU6	1	1	1	1	1
DMU7	1	1	1	1	1
DMU8	0.94324	0.931995	0.934155	0.978302	0.993742
DMU9	1	1	1	1	1

It is observed from table 2 that all DMUs are efficient except DMU2 and DMU8.

Next as in step 2, static temporal model (6) for each DMU is executed. For this, Each DMU is individually considered over all time periods from 2005-2009. The efficiencies for temporal model run are summarized in table 3. T1 represents year 2005, T2 represents year 2006 and so on T3, T4, T5 represent years 2007, 2008 and 2009 respectively.

Table 3: Temporal efficiency index for all 9 DMUs

	Temporal Efficiency Index				
DMU/TIME	TI	T2	T3	T4	T5
DMU1	1	1	1	1	1
DMU2	1	1	1	1	1
DMU3	1	1	1	1	1
DMU4	1	1	1	1	1
DMU5	1	1	1	1	1
DMU6	1	1	1	1	1
DMU7	1	1	1	1	1
DMU8	0.999165	1	1	1	1
DMU9	1	1	1	1	1

Next, as mentioned in step 3, weight matrices are formed to find range for weight restrictions. Thus, the weights so obtained for each DMU are tabulated to find optimal range of each performance measure. Some of the weight matrices are shown in table 4, 5, 6 and 7.

Table 4: weight matrix for DMU 1

DMU1	U1	U2	V1	V2	V3	V4	V5
T1	0	2.58E-04	0	0	0	2.56E-06	0
T2	0	2.56E-04		8.61E-07	0	1.77E-06	0
T3	3.08E-05	0	1.72E-06	0	0.00E+00	9.99E-07	0
T4	0	2.54E-04	3.22E-06	0	0	0.00E+00	0.00E+00
T5	0	2.53E-04	0	3.74E-06	0	0.00E+00	0

Table 5: weight matrix for DMU 4

DMU4	U1	U2	V1	V2	V3	V4	V5
T1	7.66E-06	0.00E+00	0	0	0	5.97E-07	0
T2	1.20E-06	8.10E-05	0	0.00E+0 0	9.79E-07	1.14E-07	0.00E+0 0
T3	7.27E-06	0	1.86E-07	0	0.00E+00	3.26E-07	8.02E-06
T4	6.64E-06	1.32E-05	0.00E+0 0	7.05E-07	0.00E+00	0.00E+00	0.00E+0 0
T5	0.00E+0 0	9.56E-05	8.77E-09	0.00E+0 0	0.00E+00	5.80E-07	0.00E+0 0

Table 6: weight matrix for DMU 7

DMU 7	U1	U2	V1	V2	V3	V4	V5
T1	0	1.93E-04	0	0	0	1.25E-06	0
T2	6.70E-06	1.13E-04	0	0.00E+0 0	1.29E-06	5.30E-07	0.00E+00
T3	1.57E-05	0	2.29E-07	1.40E-07	0.00E+00	0.00E+00	0.00E+00
T4	1.03E-05	7.13E-05	1.98E-06	0.00E+0 0	0.00E+00	0.00E+00	0.00E+00
T5	0	1.92E-04	2.05E-06	0.00E+00	0.00E+00	0.00E+00	0.00E+00

Table 7: weight matrix for DMU 9

DMU 9	U1	U2	V1	V2	V3	V4	V5
T1	0	1.21E-04	0	0	0	9.60E-07	0
T2	0	1.20E-04	0	2.35E-08	0	73E-07	3.64E-05
T3	1.14E-05	0	0.00E+0 0	9.87E-07	0.00E+00	41E-08	0.00E+0 0
T4	6.77E-06	98E-05	0	1.07E-06	0.00E+00	0.00E+00	0.00E+0 0
T5	0	1.19E-04	1.24E-06	0.00E+0 0	0	0.00E+00	0.00E+0 0

Next, as in step 4, we decide for windows and run the window analysis model with our weight modeling. In this example, 7 windows are taken for calculations as follows:

Windows:

1. 2005 (9 DMUs) 2. 2005-2006 (18 DMUs) 3. 2005-2006-2007 (27 DMUs) 4. 2006-2007-2008 (27 DMUs) 5. 2007-2008-2009 (27 DMUs) 6. 2008-2009 (18 DMUs) 7 2009 (9 DMUs)

The modified dynamic DEA model (13) is formulated and solved for all DMUs in all windows. Therefore, in this case, in windows 1 and 7, 9 models are run. In window 2 and 6, 18 models are run and window 3, 4 and 5, 27 models are run. Therefore, in total 135 models are formulated and run to get modified dynamic efficiency index for DMUs. All the efficiency scores thus obtained are put together in the following Dynamic efficiency matrix as given in table 8.

Table 8: Modified Dynamic efficiency matrix

	Modified Dynamic Efficiency Index						
DMU/ Year	2005	2005-06	2005-06-07	2006-07-08	2007-08-09	2008-09	2009
DMU1	1	1	1	1	1		
		1	1	1	1	1	
			1	1	1	1	1
DMU2	0.925196	0.911673	0.899973	0.906427	0.9267		
		0.9307	0.906427	0.914155	0.902619	0.918538	
			0.914155	0.895127	0.895729	0.91922	0.926998
DMU3	1	1	1	1	1		
		1	1	1	1	1	
			1	1	1	1	1
DMU4	1	1	1	1	1		
		1	0.999467	1	0.978784	1	
			1	0.979167	1	1	1
DMU5	1	0.997564	0.997564	1	1		
		1	1	1	0.996118	1	
			1	0.994094	0.994271	1	1
DMU6	1	1	1	1	1		
		1	0.99817	1	1	1	
			1	1	1	1	1
DMU7	1	0.983616	0.979845	0.994131	1		
		1	0.990251	1	1	1	
			1	1	1	1	1
DMU8	0.94324	0.927397	0.917044	0.918952	0.932831		
		0.931995	0.920345	0.925157	0.943883	0.960552	
			0.926794	0.94006	0.949388	0.979568	0.993742
MU9	1	1	1	1	1		
		1	1	1	0.977537	0.996779	

			0.999463	0.981386	1	1	1

VII. Results and discussion

Table 2 shows the static efficiency values for DMUs in different time periods. It is observed that except DMU 2 and DMU 8, rest all DMUs are efficient in all time period. This may not necessarily imply that DMUs are stably efficient since with large number of inputs and outputs in comparison to number of DMUs, discrimination power of DEA is reduced. Thus, to observe the consistency and stability in performances of DMUs, we procced to find modified dynamic efficiency index. Table 3 shows that most of the efficiency scores are 1. This is because, when a DMU is evaluated against itself over different time period using model(6), each time same DMU efficiency is maximized within constraint of its own efficiency over different time period. Weights of inputs and outputs thus obtained from this model run are used to form weight matrices. Some of these weight matrices are shown in tables 4, 5, 6, 7 representing weights of DMU1, DMU4, DMU7 and DMU9 respectively. From these tables, the optimal weight range of inputs and output weights for each DMU is calculated using equations (7)-(10). Since the DMU is evaluated with itself at various time period, it is assumed that this weight restriction will provide stability to DMUs efficiency evaluation in window analysis. Modified dynamic efficiency of DMUs using proposed modified model(13) is tabulated in table 8. From the table 8, it is observed that the DMUs 1 and 3 are consistently efficient over all windows whereas DMUs 2 and 8 are consistently inefficient over all the windows. DMU 6 is inefficient once only in window 3 and in rest windows, it is efficient. DMU 4 and DMU 7 is efficient over four windows namely windows 1, 2, 6,7 and windows 1, 5, 6, 7 respectively. DMU 5 and DMU 9 are efficient over three windows namely windows 1, 6, 7 and windows 1, 2 7 respectively.

VIII. Conclusion

In this paper, dynamic DEA models are studied and weight modeling in window analysis for multi-time period performance evaluation is proposed. To restrict weight flexibility in dynamic DEA model, the weights generated for inputs and outputs as a result of static temporal efficiency evaluation, are used in modified dynamic model. These weight restrictions are applied in different windows for efficiency evaluations. The resultant efficiencies obtained over multi-time period using modified dynamic DEA model are being observed for stability and consistency of efficiency scores. The consistent efficiency index is also defined and the applicability of proposed model shown using a numerical illustration. Consistent efficient DMUs, which are efficient over all windows are also found in the numerical example.

IX. References

Angulo-Meza, L., & Lins, M. P. E. (2002). Review of methods for increasing discrimination in data envelopment analysis. Annals of Operations Research, 116(1), 225-242.

Banker, R. D., Charnes, A., & Cooper, W. W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. Management science, 30(9), 1078-1092.

Bogetoft, P., Färe, R., Grosskopf, S., Hayes, K., & Taylor, L. (2008). Network DEA: some applications and illustrations. In Network DEA (pp. 5-12).

Chang, H., Choy, H. L., Cooper, W. W., & Ruefli, T. W. (2009). Using Malmquist Indexes to measure changes in the productivity and efficiency of US accounting firms before and after the Sarbanes–Oxley Act. Omega, 37(5), 951-960.

Charnes, A., Clark, C. T., Cooper, W. W., & Golany, B. (1983). *A developmental study of data envelopment analysis in measuring the efficiency of maintenance units in the US air forces*. TEXAS UNIV AT AUSTIN CENTER FOR CYBERNETIC STUDIES.

Charnes, A., Cooper, W., Lewin, A. Y., & Seiford, L. M. (1997). Data envelopment analysis theory, methodology and applications. *Journal of the Operational Research society*, *48*(3), 332-333.

Charnes, A., Cooper, W. W., & Rhodes, E. (1981). Evaluating program and managerial efficiency: an application of data envelopment analysis to program follow through. *Management science*, *27*(6), 668-697.

Charnes, A., Cooper, W. W., & Rhodes, E. (1979). Measuring the efficiency of decision-making units. *European journal of operational research*, *3*(4), 339-338.

Chen, C. M. (2009). A network-DEA model with new efficiency measures to incorporate the dynamic effect in production networks. *European Journal of Operational Research*, *194*(3), 687-699.

Cook, W. D., Kazakov, A., & Roll, Y. (1994). On the measurement and monitoring of relative efficiency of highway maintenance patrols. In *Data envelopment analysis: Theory, methodology, and applications* (pp. 195-210). Springer, Dordrecht.

Cooper, W. W., Seiford, L. M., & Tone, K. (2007). *Data envelopment analysis: a comprehensive text with models, applications, references and DEA-solver software* (Vol. 2, p. 489). New York: Springer.

Cooper, W. W., Seiford, L. M., & Zhu, J. (2011). Data envelopment analysis: History, models, and interpretations. In *Handbook on data envelopment analysis* (pp. 1-39). Springer, Boston, MA.

Dyson, R. G., & Thanassoulis, E. (1988). Reducing weight flexibility in data envelopment analysis. *Journal of the operational research society*, *39*(6), 563-576.

Färe, R., & Grosskopf, S. (1996). Intertemporal Production Frontiers: With Dynamic DEA Kluwer Academic Publishers.

Flokou, A., Aletras, V., & Niakas, D. (2017). A window-DEA based efficiency evaluation of the public hospital sector in Greece during the 5-year economic crisis. *PloS one*, *12*(5), e0177946.

Halkos, G. E., & Tzeremes, N. G. (2009). Exploring the existence of Kuznets curve in countries' environmental efficiency using DEA window analysis. *Ecological Economics*, *68*(7), 2168-2176.

Kao, C. (2008, July). Network data envelopment analysis: current development and future research. In *Asia-Pacific productivity conference (APPC)* (pp. 10-15).

Kazley, A. S., & Ozcan, Y. A. (2009). Electronic medical record use and efficiency: A DEA and windows analysis of hospitals. *Socio-Economic Planning Sciences*, *43*(3), 209-216.

Klopp, G. A. (1985). *The analysis of the efficiency of productive systems with multiple inputs and outputs*. University of Illinois at Chicago.

Malmquist, S. (1953). Index numbers and indifference surfaces. *Trabajos de estadística*, *4*(2), 209-242.

Nemoto, J., & Goto, M. (2003). Measurement of dynamic efficiency in production: an application of data envelopment analysis to Japanese electric utilities. *Journal of Productivity analysis*, *19*(2), 191-210.

Park, K. S., & Park, K. (2009). Measurement of multiperiod aggregative efficiency. *European journal of operational research*, *193*(2), 567-580.

Pastor, J. T., Ruiz, J. L., & Sirvent, I. (1999). An enhanced DEA Russell graph efficiency measure. *European Journal of Operational Research*, *115*(3), 596-607.

Podinovski, V. V. (2016). Optimal weights in DEA models with weight restrictions. *European Journal of Operational Research*, *254*(3), 916-924.

Sueyoshi, T., & Goto, M. (2012). Efficiency-based rank assessment for electric power industry: a combined use of data envelopment analysis (DEA) and DEA-discriminant analysis (DA). *Energy Economics*, *34*(3), 634-644.

Sueyoshi, T., & Sekitani, K. (2005). Returns to scale in dynamic DEA. *European journal of operational research*, *161*(2), 536-544.

Tone, K. (2001). A slacks-based measure of efficiency in data envelopment analysis. *European journal of operational research*, *130*(3), 498-509.

Yue, P. (1992). Data envelopment analysis and commercial bank performance: a primer with applications to Missouri banks. *IC² Institute Articles*.

***