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Control chart using bootstrap method for logistic-exponential percentiles

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***Abstract*—Lan and Leemis (2008) introduced logistic-exponential ($\mathcal{LE}$) distribution which has varied applications in lifetime modellings. In this article, we consider parametric bootstrap control charts ($\mathcal{BCC}$s) for detecting a shift in the percentile of $\mathcal{LE}$ distribution in a process monitoring situation. Four parametric $\mathcal{BCC}$s based on maximum likelihood method, method of least squares, method of Cramèr-von-Mises and method of maximum product of spacings are used for monitoring percentiles of $\mathcal{LE}$ distribution. We perform simulations to see the performances of the proposed four $\mathcal{BCC}$s with respect to average run length. Finally, one data set is analyzed to illustrate our results.**

***Index Terms*—Average run length; bootstrap control chart; classical methods of estimation; logistic-exponential distribution; logistic-exponential percentile.**

I. Introduction

One of the important tools of statistical process control (SPC) is the control chart which is primarily used for monitoring and improvement of the production process. The purpose of process monitoring techniques is to detect an unusual cause or causes to reduce the number of defective items so as to maximize the profit [see, Montgomery (2009)]. Control charts are now extensively used, not only in industry, but also in many other fields with real exertions, such as health care, packing industry, environmental sciences etc to monitor a process. A common practice to monitor control charts is that the process data come from some known probability distribution (either normal or non-normal). The usual Shewhart X bar and R control chart assume that the observed process data come from normal distribution. However, when the sampling distribution of an estimator of the parameter is not available theoretically, bootstrap methods (both parametric and non-parametric) are helpful to obtain the limits of control chart. Further, when the underlying distribution is skewed, bootstrap chart has an advantage over Shewhart-type chart because it can alarm for an out-of-control status faster than the later type chart [see, Liu and Tang (1996) and Jones and Woodall (1998) for details]. Recently, several authors have developed parametric bootstrap control chart ($\mathcal{BCC}$) to monitor percentiles for different distributions based on different methods of estimation. In this regard, readers may refer to the works of Nichols and Padgett (2005), Lio and Park (2008, 2010), Lio et al. (2014), Rezac et al. (2015), Chaing et al. (2017) and many others.

In this article, parametric $\mathcal{BCC}$s for the $\mathcal{LE}$ percentiles are obtained employing the techniques of $\mathcal{MLE}$, $\mathcal{LSE}$, $\mathcal{CME}$ and $\mathcal{MPSE}$ which are defined as $\mathcal{MLE}_B$, $\mathcal{LSE}_B$, $\mathcal{CME}_B$ and $\mathcal{MPSE}_B$, respectively. The remaining article is presented as follows: A brief introduction of the $\mathcal{LE}$ distribution is presented in Section 2. The $\mathcal{LE}$ percentiles estimates based on $\mathcal{MLE}$, $\mathcal{LSE}$, $\mathcal{CME}$ and $\mathcal{MPSE}$ are obtained in Section 3. In Section 4, parametric $\mathcal{BCC}$s of $\mathcal{MLE}_B$, $\mathcal{LSE}_B$, $\mathcal{CME}_B$, $\mathcal{MPSE}_B$ for $\mathcal{LE}$ percentiles are derived. In Section 5, the behaviour and performances of the proposed parametric $\mathcal{BCC}$s for $\mathcal{LE}$ percentiles are evaluated by using Monte Carlo simulations. One example based on failure data of Alloy $T7987$ is provided for illustration purposes in Section 6. Finally, conclusion is made in Section 7.

II. Logistic-exponential distribution

Logistic-exponential ($\mathcal{LE}$) distribution was introduced in the statistical literature by Lan and Leemis (2008). Lan and Leemis (2008) pointed out that $\mathcal{LE}$ distribution can accomodates constant, increasing, decreasing, bathtub and uni-modal failure rate shapes and since all products or items' hazard rate function exhibits at least one of the aforementioned characteristics of the hazard functions, it is useful in reliability analysis, product and process control etc. Although, it has flexible

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hazard rate shapes, very little attention has been given to different branches of statistics, like statistical quality control, reliability, survival analysis etc. The survival function of the $\mathcal{LE}$ distribution resembles the log-logistic survival function with its base changed to an exponential function, which is why it is called "$\mathcal{LE}$". The moments are finite, although they cannot be expressed in closed form. Applications of this distribution in variety of fields can be seen in Chatterjee and Singh (2014), van Staden and King (2016), Mahto et al. (2019) and Ali et al. (2020). The probability density and cumulative distribution functions of the $\mathcal{LE}$ distribution are

$$g(t;\Theta) = \frac{\lambda.\kappa\left(e^{\lambda t}-1\right)^{(\kappa-1)}e^{\lambda t}}{\left\{1+\left(e^{\lambda t}-1\right)^{\kappa}\right\}^2}\ ; \quad t>0,\ \kappa,\ \lambda>0, \quad (1)$$

$$G(t;\Theta) = \frac{\left(e^{\lambda t}-1\right)^{\kappa}}{1+\left(e^{\lambda t}-1\right)^{\kappa}}\ ; \quad t>0,\ \kappa,\ \lambda>0, \quad (2)$$

where, $\Theta=(\kappa,\lambda)$. The $100p$th percentile of the $\mathcal{LE}$ distribution and its is given by the Equations (1) and (2) can be represented as

$$Q(p;\Theta) = \frac{1}{\lambda}\log\left\{+\left(\frac{p}{1-p}\right)^{\frac{1}{\kappa}}\right\}\ ; \quad 0<p<1. \quad (3)$$

where κ and λ are the shape and scale parameters, respectively. For $\kappa>1$, the hazard rates of the distribution is unimodal shaped, for $\kappa=1$, the hazard rate of the distribution is constant and for $\kappa<1$, the hazard rates of the distribution is bathtub-shaped, respectively.

III. Estimation of $\mathcal{LE}$ percentile

This section deals with the estimation of four traditional methods, namely method of $\mathcal{MLE}$, method of $\mathcal{LSE}$, method of $\mathcal{CME}$ and method of $\mathcal{MPSE}$ to estimate $Q(p;\Theta)$ for the $\mathcal{LE}$ distribution.

$\mathcal{MLE}$

Let $\mathcal{T}=(T_1,T_2\cdots T_n)$ be a random sample of size n drawn from two parameter $\mathcal{LE}$ distribution, given in Equation (1). Thus the likelihood function can be written as follows:

$$\begin{aligned} L(\Theta) &= \prod_{i=1}^{n} g(t_i;\Theta) \\ &= \prod_{i=1}^{n}\frac{\lambda.\kappa\left(e^{\lambda t_i}-1\right)^{(\kappa-1)}e^{\lambda t_i}}{\left\{1+\left(e^{\lambda t_i}-1\right)^{\kappa}\right\}^2}. \end{aligned} \quad (4)$$

Then, the log-likelihood function can be written as

$$\log L(\Theta) = n\ln(\lambda)+n\ln(\kappa)+(\kappa-1)\sum_{i=1}^{n}\ln\left(e^{\lambda t_i}-1\right)+ \lambda\sum_{i=1}^{n}t_i-2\sum_{i=1}^{n}\ln\left\{1+1+\left(e^{\lambda t_i}-1\right)^{k}\right\}. \quad (5)$$

The $\mathcal{MLE}$s of κ and λ, say $\hat{\kappa}_{mle}$ and $\hat{\lambda}_{mle}$, respectively can be obtained as an iterative solutions of the following two equations:

$$\begin{aligned} \frac{\partial\log L(\Theta)}{\partial\kappa} &= \frac{n}{\kappa}+\sum_{i=1}^{n}\ln\left(e^{\lambda t_i}-1\right) \\ &\quad -2\sum_{i=1}^{n}\frac{\left(e^{\lambda t_i}-1\right)^{k}\ln\left(e^{\lambda t_i}-1\right)}{1+\left(e^{\lambda t_i}-1\right)^{\kappa}}, \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{\partial\log L(\Theta)}{\partial\lambda} &= \frac{n}{\lambda}+\sum_{i=1}^{n}\frac{(\kappa-1)t_i.e^{\lambda t_i}}{e^{\lambda t_i}-1}+\sum_{i=1}^{n}t_i \\ &\quad -2\sum_{i=1}^{n}\frac{\kappa\left(e^{\lambda t_i}-1\right)^{k-1}t_i.e^{\lambda x_i}}{1+\left(e^{\lambda t_i}-1\right)^{\kappa}}. \end{aligned} \quad (7)$$

To obtain the $\mathcal{MLE}$s, an optimization technique can be employed to obtain the solutions of the Equations (6) and (7). Here, non-linear minimization ($\mathcal{NLM}$) [see, Dennis and Schnabel (1983)] method is used to obtain the estimates of the parameters of the $\mathcal{LE}$ distribution. For $\mathcal{NLM}$ method, we have to iterate the negative log-likelihood function using some starting guess value for the parameters, say, moment estimates of κ and λ, and get the estimates of κ and λ as $\hat{\kappa}_{mle}$ and $\hat{\lambda}_{mle}$, respectively. Replacing Θ with $\hat{\Theta}_{mle}$ in Equation (3), the $\mathcal{MLE}$ of $Q(p;\Theta)$ can be obtained as

$$\hat{Q}(p;\hat{\Theta})_{mle} = \frac{1}{\hat{\lambda}_{mle}}\log\left\{+\left(\frac{p}{1-p}\right)^{\frac{1}{\hat{\kappa}_{mle}}}\right\} \text{ for } 0<p<1. \quad (8)$$

$\mathcal{LSE}$

Suppose $t_{(1:n)}<t_{(2:n)}<\cdots<t_{(n:n)}$ of size n be the ordered random variables from a distribution function $F(t_{(i:n)};\Theta)$. Therefore, $\mathcal{LSE}$s of κ and λ, say $\hat{\kappa}_{lse}$ and $\hat{\lambda}_{lse}$ can be obtained by minimizing the following function

$$L(\kappa,\lambda)=\sum_{i=1}^{n}\left[G(t_{(i:n)};\Theta)-\frac{i}{n+1}\right]^2$$

with respect to κ and λ. Equivalently, they can be obtained by solving the following equations

$$\sum_{i=1}^{n}\left[G(t_{(i:n)};\Theta)-\frac{i}{n+1}\right]\kappa_1\left(t_{(i:n)};\Theta\right)=0,$$
$$\sum_{i=1}^{n}\left[G(t_{(i:n)};\Theta)-\frac{i}{n+1}\right]\kappa_2\left(t_{(i:n)};\Theta\right)=0,$$

where

$$\begin{aligned} \kappa_1\left(t_{(i:n)};\Theta\right) &= \frac{\partial G(t_{(i:n)};\Theta)}{\partial\kappa} \\ &= \frac{\left(e^{\lambda t_i}-1\right)^{\kappa}\log(e^{\lambda t_i}-1)}{\left[1+\left(e^{\lambda t_i}-1\right)^{\kappa}\right]^2}, \end{aligned} \quad (9)$$

and

$$\begin{aligned} \kappa_2\left(t_{(i:n)};\Theta\right) &= \frac{\partial G(t_{(i:n)};\Theta)}{\partial\lambda} \\ &= \frac{\kappa\, t_i\, e^{\lambda t_i}\left(e^{\lambda t_i}-1\right)^{k-1}}{\left[1+\left(e^{\lambda t_i}-1\right)^{k}\right]^2} \end{aligned} \quad (10)$$

respectively. Substituting the $\mathcal{LSE}$s in Eqn. (3), we can get the estimator as

$$\hat{Q}(p;\hat{\Theta})_{lse} = \frac{1}{\hat{\lambda}_{lse}}\log\left\{+\left(\frac{p}{1-p}\right)^{\frac{1}{\hat{\kappa}_{lse}}}\right\} \text{ for } 0<p<1. \quad (11)$$

$\mathcal{CME}$

By minimizing the following function, we can get the Cramér-von-Mises estimates of the unknown parameters κ and λ, say $\hat{\kappa}_{cme}$ and $\hat{\lambda}_{cme}$

$$C(\Theta) = \frac{1}{12n} + \sum_{i=1}^{n}\left[G(t_{(i:n)};\Theta) - \frac{2i-1}{2n}\right]^2$$

These estimators can also be obtained from the following non-linear equations:

$$\sum_{i=1}^{n}\left[G(t_{(i:n)};\Theta) - \frac{2i-1}{2n}\right]\kappa_1\left(t_{(i:n)};\Theta\right) = 0,$$
$$\sum_{i=1}^{n}\left[G(t_{(i:n)};\Theta) - \frac{2i-1}{2n}\right]\kappa_2\left(t_{(i:n)};\Theta\right) = 0,$$

where $\kappa_1(\cdot;\Theta)$ and $\kappa_2(\cdot;\Theta)$ are defined in Eqns. (9) and (10), respectively. Substituting the $\mathcal{CME}$s in Eqn. (3), we can get the estimator as

$$\hat{Q}(p;\hat{\Theta})_{cme} = \frac{1}{\hat{\lambda}_{cme}}\log\left\{+\left(\frac{p}{1-p}\right)^{\frac{1}{\hat{\kappa}_{cme}}}\right\} \text{ for } 0<p<1. \quad (12)$$

$\mathcal{MPSE}$

The $\mathcal{MPS}$ mathod was developed by Cheng and Amin (1983) and showed that this method is as competent as the $\mathcal{MLE}$s. Based on a random sample of size n from the $\mathcal{LE}$ distribution, the uniform spacings can be defined as follows

$$D_i(\Theta) = G(t_{(i:n)};\Theta) - G(t_{(i-1:n)};\Theta), i = 1, 2, \ldots, n+1,$$

The following function is maximized to obtain the $\mathcal{MPSE}$s of κ and λ

$$\mathcal{G}(\Theta) = \left[\prod_{i=1}^{n+1} D_i(\Theta)\right]^{\frac{1}{n+1}},$$

or, equivalently maximizing the following function

$$H(\Theta) = \frac{1}{n+1}\sum_{i=1}^{n+1}\log D_i(\Theta).$$

$\mathcal{MPSE}$s denoted as $\hat{\kappa}_{mpse}$ and $\hat{\lambda}_{mpse}$ can be obtained by solving the following nonlinear equations

$$\frac{\partial}{\partial\kappa}H(\Theta) = \frac{1}{n+1}\sum_{i=1}^{n+1}\frac{1}{D_i(\Theta)}\left[\kappa_1\left(t_{(i:n)};\Theta\right) - \kappa_1\left(t_{(i-1:n)};\Theta\right)\right] = 0,$$
$$\frac{\partial}{\partial\lambda}H(\Theta) = \frac{1}{n+1}\sum_{i=1}^{n+1}\frac{1}{D_i(\Theta)}\left[\kappa_2\left(t_{(i:n)};\Theta\right) - \kappa_2\left(t_{(i-1:n)};\Theta\right)\right] = 0.$$

where $\kappa_1(\cdot;\Theta)$ and $\kappa_2(\cdot;\Theta)$ are defined in Eqns. (9) and (10), respectively. Substituting the $\mathcal{MPSE}$s in Eqn. (3), we can get the estimator as

$$\hat{Q}(p;\hat{\Theta})_{mpse} = \frac{1}{\hat{\lambda}_{mpse}}\log\left\{+\left(\frac{p}{1-p}\right)^{\frac{1}{\hat{\kappa}_{mpse}}}\right\} \text{ for } 0<p<1 \quad (13)$$

IV. Parametric bootstrap control charts

In this section, we develop parametric bootstrap control chart for the $\mathcal{LE}$ percentiles as of the sampling distribution of statistic for the $\mathcal{LE}$ percentile is not in hand. To develop $\mathcal{BCC}$ for the $\mathcal{LE}$ percentiles, below the algorithm is provided based on $\mathcal{MLE}$s of $\Theta = (\kappa, \lambda)$.

1) If the process is stationary and under control, take $\mathcal{K}$ stochastic samples of each of size n_j ($j = 1, 2, \cdots, \mathcal{K}$) randomly taken from an $\mathcal{LE}$ distribution for unknown parameters κ and λ. We identify the measurements of the j th value by x_{ij} ($i = 1, 2, \cdots, n_j$).
2) Using the $\mathcal{MLE}$s given in Equations (3.6) and (3.7), evaluate the $\mathcal{MLE}$s of $\hat{\Theta} = (\hat{\kappa}, \hat{\lambda})$ with the combined observations of size $n = \sum_{\mathcal{J}=1}^{\mathcal{K}} n_j$.
3) Create a bootstrap values of size m, $x_1^*, x_2^*, \cdots, x_{\mathcal{M}}^*$ from the $\mathcal{LE}$ using the $\mathcal{MLE}$s obtained in Step 2. Here, $\mathcal{M}$ is the sample size which can be obtained for forthcoming subgroup values.
4) Using bootstrap sample obtained in Step 3 and obtain the $\mathcal{MLE}$s, $\hat{\kappa}^*_{mle}$ and $\hat{\lambda}^*_{mle}$.
5) For bootstrap subgroup sample obtained in Step 3 and $\hat{\Theta}^*_{mle} = (\hat{\kappa}^*, \hat{\lambda}^*)$ in Step 4, find the bootstrap estimate, $\hat{Q}^*(p;\hat{\Theta}^*_{mle})$ of the $100p$th percentile $\hat{Q}(p;\hat{\Theta})_{mle}$, where
$$\hat{Q}^*(p;\hat{\Theta})^*_{mle} = \frac{1}{\hat{\lambda}^*_{mle}}\log[1+(\frac{p}{1-p})^{\frac{1}{\hat{\kappa}^*_{mle}}}] \text{ for } 0<p<1.$$
6) Repeat Steps 3-5, quite a large number of times say, $\mathcal{B}$, to obtain $\mathcal{B}$ bootstrap estimates of $\hat{Q}(p;\hat{\Theta})_{mle}$, denoted as $\left\{\hat{Q}^{*(\mathcal{J})}(p;\hat{\Theta})^*_{mle};\ \mathcal{J} = 1, 2, \cdots, \mathcal{B}\right\}$.
7) Using the $\mathcal{B}$ bootstrap estimates obtained in Step 6, next obtain the $100(\zeta/2)$th and $100(1-\zeta/2)$th percentiles. Here, ζ is the probability that a value which is regarded as out of control when the process is assumed as in control, i.e., false alarm rate ($\mathcal{FAR}$). Thus $100(\zeta/2)$th and $100(1-\zeta/2)$th percentiles are the lower control limit $\mathcal{LCL}$ and the upper control limit $\mathcal{UCL}$ towards the ($\mathcal{BCC}$) of FAR $= \zeta$, respectively. It is understood that various interpretations of the observed percentiles have been proposed in the statistics literature. Let,
$$\bar{\hat{Q}}^*(p;\hat{\Theta})^*_{mle} = \frac{1}{\mathcal{B}}\sum_{\mathcal{J}}^{\mathcal{B}}\hat{Q}^{*(\mathcal{J})}(p;\hat{\Theta})^*_{mle}$$
and $\hat{Q}^{*(\tau)}(p;\hat{\Theta})^*_{mle}$ be the τ percentile of $\left\{\hat{Q}^{*(\mathcal{J})}(p;\hat{\Theta})^*_{mle};\ \mathcal{J} = 1, 2, \cdots, \mathcal{B}\right\}$, i.e., $\hat{Q}^{*(\tau)}(p;\hat{\Theta})^*_{mle}$ is such that
$$\frac{1}{\mathcal{B}}\sum_{\mathcal{J}=1}^{\mathcal{B}} In\left(\hat{Q}^{*(\mathcal{J})}(p;\hat{\Theta})^*_{mle} \leq \hat{Q}^{*(\tau)}(p;\hat{\Theta})^*_{mle}\right) = \tau;\ 0<\tau<1,$$
where $In(.)$ is the indicator function. Hence, $\mathcal{UCL}$, control limit ($\mathcal{CL}$), $\mathcal{LCL}$ are as follows:
$$\mathcal{UCL} = \hat{Q}^{*(\mathcal{B}\times 100(1-\zeta/2))}(p;\hat{\Theta})^*_{mle}$$
$$\mathcal{CL} = \bar{\hat{Q}}^*(p;\hat{\Theta})^*_{mle}$$
$$\mathcal{LCL} = \hat{Q}^{*(\mathcal{B}\times 100(\zeta/2))}(p;\hat{\Theta})^*_{mle}$$

Now, if the method of $\mathcal{ML}$ is replaced by the method of $\mathcal{LS}$, method of $\mathcal{CM}$ and method of $\mathcal{MPS}$ for the steps 1-3, then we get the bootstrap estimates $(\hat{\kappa}^*_{lse}, \hat{\lambda}^*_{lse})$, $(\hat{\kappa}^*_{cme}, \hat{\lambda}^*_{cme})$, $(\hat{\kappa}^*_{mpse}, \hat{\lambda}^*_{mpse})$, respectively. Hence, the plot statistics for $\mathcal{MLE}$, $\mathcal{LSE}$, $\mathcal{CME}$ and $\mathcal{MPSE}$ are $\hat{Q}^*(p;\hat{\Theta})^*_{mle}$, $\hat{Q}^*(p;\hat{\Theta})^*_{lse}$, $\hat{Q}^*(p;\hat{\Theta})^*_{cme}$ and $\hat{Q}^*(p;\hat{\Theta})^*_{mpse}$ respectively. After the completion of construction of $\mathcal{LCL}$ and $\mathcal{UCL}$ of a control chart in Phase I, the monitoring process continues in Phase II with the drawing of future samples each of size $\mathcal{M}$ from $\mathcal{LE}$ distribution and calculation of plot statistic, respectively. In case the plot statistic is between $\mathcal{LCL}$ and $\mathcal{UCL}$, the process is said to be in control, otherwise the control chart indicates for out-of-control.

V. Simulation and discussion

In this section, Monte Carlo simulation study is conducted to check the performance of the $\mathcal{LE}$ percentile $\mathcal{BCC}$s ($\mathcal{MLE}$, $\mathcal{LSE}$, $\mathcal{CME}$, $\mathcal{MPSE}$) using R package, an open-source developed by Ihaka and Gentleman (1996). Now, the under control nominal average length ($\mathcal{ARL}$) is defined by $\text{ARL}_{H_0} = 1/\gamma$ for every committed false alarm rate ($\mathcal{FAR}$) γ. The performances of the $\mathcal{LE}$ percentile control charts are examined based on simulated in-control $\mathcal{ARL}_{H_0}$ and simulated out-of-control average run length $\mathcal{ARL}_{H_1}$ and their corresponding standard errors of run lengths ($\mathcal{SERL}$s) respectively. Also, the behaviour of the $\mathcal{BCC}$s for $\mathcal{LE}$ percentiles are assessed by obtaining the average $\mathcal{LCL}$ and $\mathcal{UCL}$ and their associated standard errors ($\mathcal{SE}$s) from simulations.

For the present investigation, the $\mathcal{LE}$ distribution with $\kappa = 4.31359$ and $\lambda = 0.38756$, which is connected with the real life application is considered in next section. The simulation study is carried out with subgroup size $\mathcal{M} = 5$, subgroup number $\mathcal{K} = 20$, and different levels of $\mathcal{FAR}$s $\gamma_0 = 0.01, 0.05, 0.0027$. The given γ's are known as Type-I errors for real life applications in quality control. The chart constants of bootstrap charts are computed by using $\mathcal{B} = 5,000$ bootstrap observations. For each simulation run, the $\mathcal{ARL}$, this can be determined as the number of subgroup needed to the first out-of-control observed, and corresponding $\mathcal{SERL}$s are obtained. The average $\mathcal{LCL}$, $\mathcal{UCL}$ and corresponding $\mathcal{SE}$s are also obtained based on $5,000$ bootstrap samples. Due to page constraint, displayed some portion of results for the $\mathcal{M} = 5$, $\mathcal{K} = 20$ and $p = 0.01, 0.05, 0.10$) are given in Tables 1-4 for $\mathcal{MLE}$, $\mathcal{LSE}$, $\mathcal{CME}$ and $\mathcal{MPSE}$. Most of the truncated life tests are designed based on the mean lifetime. On the other hand, in numerous applications in industry, the small percentile of lifetime is needed to convene engineering intend purpose. Lio et al. (2010) suggested that the mean is not a good index to grab the change on the quality of the manufactured goods lifetime. For instance, products would be accepted due to a little decrease in the mean lifetime after inspection. However the smaller lifetime percentile would be significantly below the consumers anticipation. Tables 1-4 show the simulated $\mathcal{ARL}_{H_0}$ and the corresponding $\mathcal{SERL}$s for the $\mathcal{MLE}_B$, $\mathcal{LSE}_B$, $\mathcal{CME}_B$ and $\mathcal{MPSE}_B$, respectively. For $\gamma_0 = 0.01, 0.05, 0.0027$, all the $\mathcal{MLE}_B$, $\mathcal{LSE}_B$, $\mathcal{CME}_B$ and $\mathcal{MPSE}_B$ are competitive irrespective of the sample size and $\mathcal{P}$ values, excluding at $\mathcal{CME}_B$ has the simulated ARL_{H_0} smaller as compared to $\gamma_0 = 0.10, 0.05$. For $\gamma_0 = 0.0027$, $\mathcal{CME}_B$ has the simulated $\mathcal{ARL}_{H_0}$ greater than their counter part $\gamma_0 = 0.0027$. The associated $\mathcal{SERL}$s in Tables 1-4 notice that the simulated $5,000$ run lengths are coherent for all control chart. Comprehensively, the $\mathcal{MLE}_B$ and $\mathcal{MPSE}_B$ out perform the other bootstrap charts.

The average $\mathcal{UCL}$ and $\mathcal{LCL}$ and the associated $\mathcal{SE}$s are determined as specified below: for every set of $\mathcal{K}$ individual observations of size n ($\mathcal{K} = 20$ observations of the same size were used from the simulations) computed from a $\mathcal{LE}$ distribution with a given shape and scale parameters κ and λ, as explained in Step 1, Steps $2-7$ were then accomplished with $\mathcal{B} = 5,000$ to bring forth control limits. The total procedure (Steps $1-7$) was recapitulated $5,000$ times and the average $\mathcal{LCL}$s and $\mathcal{UCL}$s were determined based on $5,000$ recapitulated values of $\mathcal{LCL}$s and $\mathcal{UCL}$s, respectively. The adhering $\mathcal{SE}$s of the control limits were also determined from the corresponding $5,000$ values. A few simulation results of the average $\mathcal{LCL}$ and $\mathcal{UCL}$ with the agreeing $\mathcal{SE}$s are provided at Tables 1-4 with sample size 5 for $\mathcal{MLE}_B$, $\mathcal{LSE}_B$, $\mathcal{CME}_B$ and $\mathcal{MPSE}_B$ of percentile estimates. From Tables 1-4 noticed that the standard deviations are modest relative to the representing control limits. It has been also observed that the simulated coefficient of variations ($\mathcal{CV}$s) are less than 0.0084 for all the control limits of $\mathcal{MLE}_B$, tinier than 0.0087 for all the control limits of $\mathcal{MPSE}_B$, tinier than 0.0097 for all the control limits of $\mathcal{LSE}_B$ and tinier than 0.0127 for all the control limits of $\mathcal{CME}_B$. Hence, the proposed constructing procedure for $\mathcal{MLE}_B$ and $\mathcal{MPSE}_B$ can render stable control limits and furnish helpful advising to develop control chart in the real scenario. Since $\mathcal{MLE}_B$ and $\mathcal{MPSE}_B$ shows improve performance based on simulated $\mathcal{ARL}_{H_0}$ than the other studied control charts, $\mathcal{MLE}_B$ and $\mathcal{MPSE}_B$ will be considered for further interrogation to examine the $\mathcal{ARL}_{H_1}$ for out of control process. Control limits of $\mathcal{MLE}_B$, $\mathcal{LSE}_B$, $\mathcal{CME}_B$ and $\mathcal{MPSE}_B$ are developed with phase I samples. Hence, next observations are produced from beyond control process. The beyond control observations are then applied to calculate $\mathcal{ARL}_{H_1}$ and the representing $\mathcal{SERL}$. One can implemented beyond control mechanism in the below manner: Assume λ_0 of the in control process be constant, also κ decrease from κ_0 to for the under control process to a tinier value κ_1 for the beyond control process and can observed that which one is more sensitive to supervise a descending shift of the $\mathcal{LE}$ percentiles. Actually, this situation is also discussed in the next section with the help of a numerical example.

VI. Application

Here, we consider an example comprises to 67 specimens of Alloy $T7987$ that failed before having accumulated 300 thousand cycles of testing and was initially considered by Raqab et al. (2017). The dataset is fitted with the $\mathcal{LE}$

distribution and the $\mathcal{MLE}$s for the parameters κ and λ are 4.31 and 0.39 respectively for which the p value of one sample Kolmogorav-Smirnov ($\mathcal{KS}$) statistic is 0.5814. We generated $\mathcal{K} = 20$ under control Phase-I observations each of size $\mathcal{M} = 5$ form $\mathcal{LE}$ distribution with $\kappa_0 = 4.31$ and $\lambda_0 = 0.39$. All the 20 samples of size 5 are given in Table 6.

Let the industrial quality officers are considered with the lifetime quality of cycles of testing of the tenth percentile, $Q(p = 0.10; \Theta_0 = (\kappa_0 = 4.31, \lambda_0 = 0.39)) = 1.21$ and the process is considered as beyond control state in Phase II with a shift of the parameters κ_0 to $\kappa_1 = 3.51$ when $\lambda_0 = 0.39$. Table 7 displays 20 out of control Phase II samples. The $\mathcal{MLE}_B$ chart was demonstrated using Table VI with $\gamma = 0.0027$ and $\mathcal{B} = 5,000$. The bootstrap chart control limits based on $\mathcal{MLE}$s are obtained by

$$\mathcal{UCL} = 2.452$$

$$\mathcal{CL} = 1.324$$

$$\mathcal{LCL} = 0.871$$

From Figure 1, it is observed that the $\mathcal{MLE}_B$ gives asymmetric control limits the $\mathcal{CL}$ and display statistic of sample percentiles at $p = 0.10$ at sample 7 is close to $\mathcal{LCL}$, but it did not show out of control. It is signal out first at the sample point 52. As a matter of fact that except sample 52, all the sample statistics plotted in Figure 1 are lie between the lower and upper control limits. The proposed parametric $\mathcal{BCC}$s appears as pretty powerful and can be well carried out for the case where one necessarily to follow $\mathcal{LE}$ distribution for the probability of process measurements. To heighten the defensiveness of the $\mathcal{MLE}_B$ to alarm an out-of-control, we suggested to look at employ 25 Phase I observations entirely for size 6 or apply 20 Phase I observations entirely for size 8 to develop the $\mathcal{MLE}_B$.

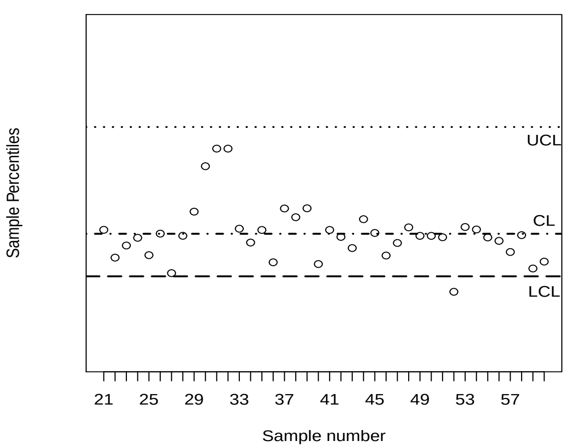

Fig. 1. $\mathcal{MLE}_B$ control chart for the Phase-II data set when $\gamma = 0.0027$.

VII. Conclusions

The parametric $\mathcal{BCC}$s based on the different classical methods of estimation, viz., $\mathcal{MLE}$, $\mathcal{LSE}$, $\mathcal{CME}$ and $\mathcal{MPSE}$, respectively are developed to supervise the percentiles of the $\mathcal{LE}$ distribution. The sampling distribution of the statistic for the $\mathcal{LE}$ percentile is not available. Thus, the conventional Shewhart control chart $\mathcal{SCC}$ for the $\mathcal{LE}$ distribution is not tractable. Therefore, in this article, we propose to use $\mathcal{BCC}$ in order to deal with the issues described. Using extensive Monte Carlo simulations, we established that the $\mathcal{ARL}$ for $\mathcal{MLE}_B$ chart provide good results for $\mathcal{LE}$ percentiles by the developed parametric bootstrap approach. The sampling distribution of the predicted statistic for the $\mathcal{LE}$ percentile is generally asymmetric and is not suitable for small sample numbers, hence the parametric $\mathcal{BCC}$ should be used instead. Therefore, it is advised to monitor the $\mathcal{LE}$ percentiles in practise using the $\mathcal{MLE}_B$. We can take into account additional observations in each sample for Phase I to produce the control chart, which will enhance the $\mathcal{MLE}_B$'s capacity to warn a signal that is outside of the acceptable range. In further study, the suggested approach can be expanded to a variety of lifetime distributions.

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TABLE I
IN-CONTROL $\mathcal{ARL}$ ESTIMATE, CORRESPONDING $\mathcal{SERL}$, AVERAGE $\mathcal{LCL}$, $\mathcal{UCL}$ AND CORRESPONDING $\mathcal{SE}$S FOR $\mathcal{MLE}_B$ CHART FOR $\gamma_0 = 0.10, 0.01, 0.0027$ $\mathcal{FAR}$S.

Percentile	$\mathcal{K} = 20$, $\mathcal{M} = 5$		$\kappa = 4.31359$		$\lambda = 0.38756$	
	$\mathcal{ARL}$	$\mathcal{SERL}$	$\mathcal{LCL}$	$\mathcal{SE}$	$\mathcal{UCL}$	$\mathcal{SE}$
	γ_0=0.10 ($\mathcal{FAR}$)				$\mathcal{ARL}_{H_0} = 10$	
$p = 0.01$	10.0036	0.1328	0.5871	0.0322	2.8973	0.0521
$p = 0.05$	10.0753	0.1377	0.6122	0.0345	2.9122	0.0537
$p = 0.10$	10.1128	0.1397	0.6348	0.0382	2.9366	0.0543
$p = 0.25$	10.2652	0.1425	0.6893	0.0395	2.9762	0.0577
	γ_0=0.01 ($\mathcal{FAR}$)				$\mathcal{ARL}_{H_0} = 100$	
$p = 0.01$	103.3211	1.6892	0.5346	0.0311	3.1211	0.0657
$p = 0.05$	108.7862	1.8123	0.5732	0.0327	3.2342	0.0662
$p = 0.10$	113.2237	1.9789	0.5984	0.0334	3.4126	0.0671
$p = 0.25$	119.3872	2.1136	0.6233	0.0346	3.4653	0.0683
	γ_0=0.0027 ($\mathcal{FAR}$)				$\mathcal{ARL}_{H_0} = 370.37$	
$p = 0.01$	416.2244	1.9768	0.4763	0.0304	3.2214	0.0677
$p = 0.05$	419.2981	1.9989	0.4983	0.0309	3.2563	0.0689
$p = 0.10$	423.7769	2.1327	0.5322	0.0314	3.4987	0.0694
$p = 0.25$	434.1174	2.2249	0.5553	0.0321	3.5436	0.0711

TABLE II
IN-CONTROL $\mathcal{ARL}$ ESTIMATE, CORRESPONDING $\mathcal{SERL}$, AVERAGE $\mathcal{LCL}$, $\mathcal{UCL}$ AND CORRESPONDING $\mathcal{SE}$S FOR $\mathcal{LSE}_B$ CHART FOR $\gamma_0 = 0.10, 0.01, 0.0027$ $\mathcal{FAR}$S.

Percentile	$\mathcal{K} = 20$, $\mathcal{M} = 5$		$\kappa = 4.31359$		$\lambda = 0.38756$	
	$\mathcal{ARL}$	$\mathcal{SERL}$	$\mathcal{LCL}$	$\mathcal{SE}$	$\mathcal{UCL}$	$\mathcal{SE}$
	γ_0=0.10 ($\mathcal{FAR}$)				$\mathcal{ARL}_{H_0} = 10$	
$p = 0.01$	10.0006	0.1287	0.5237	0.0313	2.9347	0.0566
$p = 0.05$	10.0022	0.1321	0.5732	0.0319	2.9983	0.0573
$p = 0.10$	10.0759	0.1389	0.6089	0.0327	3.0729	0.0589
$p = 0.25$	10.1127	0.1422	0.6322	0.0335	3.1123	0.0613
	γ_0=0.01 ($\mathcal{FAR}$)				$\mathcal{ARL}_{H_0} = 100$	
$p = 0.01$	100.0764	1.3233	0.5113	0.0296	3.1749	0.0699
$p = 0.05$	100.8832	1.6679	0.5423	0.0311	3.2238	0.0714
$p = 0.10$	103.0914	1.9348	0.5783	0.0321	3.2981	0.0722
$p = 0.25$	106.1127	2.0237	0.6127	0.0330	3.3318	0.0732
	γ_0=0.0027 ($\mathcal{FAR}$)				$\mathcal{ARL}_{H_0} = 370.37$	
$p = 0.01$	407.3754	1.5237	0.4234	0.0293	3.3321	0.0724
$p = 0.05$	410.7764	1.7866	0.4673	0.0308	3.4612	0.0733
$p = 0.10$	414.2217	2.0978	0.4982	0.0317	3.5211	0.0747
$p = 0.25$	419.8876	2.6893	0.5359	0.0327	3.5982	0.0766

TABLE III
IN-CONTROL $\mathcal{ARL}$ ESTIMATE, CORRESPONDING $\mathcal{SERL}$, AVERAGE $\mathcal{LCL}$, $\mathcal{UCL}$ AND CORRESPONDING $\mathcal{SE}$S FOR $\mathcal{CME}_B$ CHART FOR $\gamma_0 = 0.10, 0.01, 0.0027$ $\mathcal{FAR}$S.

	$\mathcal{K}=20$, $\mathcal{M}=5$		$\kappa = 4.31359$		$\lambda = 0.38756$	
Percentile	$\mathcal{ARL}$	$\mathcal{SERL}$	$\mathcal{LCL}$	$\mathcal{SE}$	$\mathcal{UCL}$	$\mathcal{SE}$
	γ_0=0.10 ($\mathcal{FAR}$)				$\mathcal{ARL}_{H_0} = 10$	
$p = 0.01$	9.1356	0.1223	0.5188	0.0309	2.9961	0.0611
$p = 0.05$	9.5022	0.1311	0.5632	0.0321	3.1427	0.0626
$p = 0.10$	10.0049	0.1359	0.5976	0.0332	3.2671	0.0637
$p = 0.25$	10.1211	0.1453	0.6322	0.0347	3.3246	0.0645
	γ_0=0.01 ($\mathcal{FAR}$)				$\mathcal{ARL}_{H_0} = 100$	
$p = 0.01$	99.2316	1.3444	0.4998	0.0288	3.1929	0.0717
$p = 0.05$	99.8752	1.5632	0.5327	0.0307	3.2893	0.0732
$p = 0.10$	102.0722	1.8746	0.5876	0.0321	3.3879	0.0743
$p = 0.25$	104.2465	1.8978	0.6457	0.0334	3.4562	0.0766
	γ_0=0.0027 ($\mathcal{FAR}$)				$\mathcal{ARL}_{H_0} = 370.37$	
$p = 0.01$	404.2465	1.4356	0.4089	0.0288	3.4327	0.0755
$p = 0.05$	406.3576	1.6782	0.4239	0.0297	3.5489	0.0769
$p = 0.10$	408.4982	1.8978	0.4892	0.0312	3.6123	0.0786
$p = 0.25$	411.7679	2.0861	0.5348	0.0326	3.6987	0.0798

TABLE IV
IN-CONTROL $\mathcal{ARL}$ ESTIMATE, CORRESPONDING $\mathcal{SERL}$, AVERAGE $\mathcal{LCL}$, $\mathcal{UCL}$ AND CORRESPONDING $\mathcal{SE}$S FOR $\mathcal{MPSE}_B$ CHART FOR $\gamma_0 = 0.10, 0.01, 0.0027$ $\mathcal{FAR}$S.

	$\mathcal{K}=20$, $\mathcal{M}=5$		$\kappa = 4.31359$		$\lambda = 0.38756$	
Percentile	$\mathcal{ARL}$	$\mathcal{SERL}$	$\mathcal{LCL}$	$\mathcal{SE}$	$\mathcal{UCL}$	$\mathcal{SE}$
	γ_0=0.10 ($\mathcal{FAR}$)				$\mathcal{ARL}_{H_0} = 10$	
$p = 0.01$	10.0043	0.1344	0.5632	0.0317	2.9027	0.0524
$p = 0.05$	10.0766	0.1389	0.5832	0.0321	2.9237	0.0528
$p = 0.10$	10.1138	0.1407	0.6123	0.0327	2.9567	0.0533
$p = 0.25$	10.2657	0.1436	0.6245	0.0333	2.9946	0.0538
	γ_0=0.01 ($\mathcal{FAR}$)				$\mathcal{ARL}_{H_0} = 100$	
$p = 0.01$	103.3216	1.6911	0.5233	0.0303	3.1428	0.0681
$p = 0.05$	108.7874	1.8184	0.5517	0.0309	3.1783	0.0694
$p = 0.10$	113.2248	1.9832	0.5843	0.0314	3.2239	0.0699
$p = 0.25$	119.3891	2.1177	0.5987	0.0321	3.2763	0.0712
	γ_0=0.0027 ($\mathcal{FAR}$)				$\mathcal{ARL}_{H_0} = 370.37$	
$p = 0.01$	416.4572	1.9981	0.4544	0.0301	3.2967	0.0711
$p = 0.05$	419.3237	1.9992	0.4862	0.0309	3.3256	0.0718
$p = 0.10$	423.8793	2.1357	0.5127	0.0317	3.3891	0.0726
$p = 0.25$	434.1321	2.2281	0.5438	0.0324	3.4312	0.0733

TABLE V
TOP 20 SUBGROUPS GENERATED FROM LE DISTRIBUTION WITH $\kappa_0 = 4.31, \lambda_0 = 0.39$.

Subgroup number	Generated data set					Subgroup number	Generated data set				
1	2.14	2.26	1.97	1.57	1.43	11	1.93	1.96	2.13	1.99	1.61
2	1.46	2.50	2.62	0.77	1.50	12	1.49	3.41	1.82	1.49	1.93
3	1.96	2.39	2.21	2.16	2.15	13	0.90	1.88	2.03	1.43	1.28
4	2.14	2.32	2.58	2.58	1.66	14	2.30	2.21	1.84	1.83	1.66
5	1.94	2.52	2.30	2.18	2.10	15	1.32	1.89	1.52	1.50	1.29
6	1.20	2.01	1.62	2.15	1.66	16	1.43	2.79	2.15	2.26	1.91
7	2.95	2.05	2.22	1.47	2.49	17	2.71	2.07	0.97	2.64	1.55
8	1.81	2.69	2.12	1.36	0.82	18	2.65	1.73	1.82	1.84	1.07
9	1.60	1.35	2.02	1.81	1.73	19	2.69	1.28	1.85	1.85	1.59
10	1.96	1.68	1.06	1.33	1.61	20	1.75	1.35	2.83	1.63	1.41

TABLE VI
FORTY OUT-OF-CONTROL SUBGROUPS GENERATED FROM LE DISTRIBUTION WITH $\kappa_1 = 3.51, \lambda_0 = 0.39$.

Subgroup number	Generated data set					Subgroup number	Generated data		
21	1.52	1.94	1.21	1.73	2.30	41	2.10	1.51	1.69
22	1.53	0.79	1.31	1.40	1.39	42	1.89	0.72	2.01
23	2.07	1.22	2.48	1.41	1.29	43	1.02	2.37	1.42
24	4.26	1.51	1.43	1.85	1.81	44	1.68	2.30	1.71
25	1.31	1.61	1.12	2.00	1.17	45	2.04	1.25	1.39
26	2.10	2.92	1.17	1.36	2.00	46	1.55	2.70	1.08
27	1.65	0.53	2.53	0.99	2.47	47	1.12	2.44	2.68
28	1.62	1.09	1.50	2.07	1.87	48	1.87	1.74	1.71
29	1.47	2.97	1.69	2.50	1.94	49	1.79	1.62	0.93
30	2.89	2.83	2.39	1.73	2.38	50	1.85	1.77	1.59
31	2.19	2.29	2.87	2.70	2.54	51	2.32	1.50	1.20
32	0.99	1.19	0.94	1.68	2.36	52	0.20	1.69	1.50
33	1.35	2.14	2.95	1.30	2.04	53	2.38	1.25	2.16
34	1.43	1.47	1.90	1.06	1.92	54	2.11	1.23	1.26
35	1.46	1.70	1.22	1.82	1.95	55	2.10	1.03	2.33
36	1.48	1.18	1.60	1.29	0.83	56	1.62	1.27	1.57
37	1.47	1.96	2.19	2.12	1.66	57	1.49	0.99	1.21
38	1.39	3.04	2.20	1.78	1.85	58	3.03	1.96	0.83
39	1.49	2.55	1.98	1.80	1.97	59	2.64	1.25	0.41
40	1.74	1.21	1.91	1.91	3.40	60	1.03	1.68	3.03